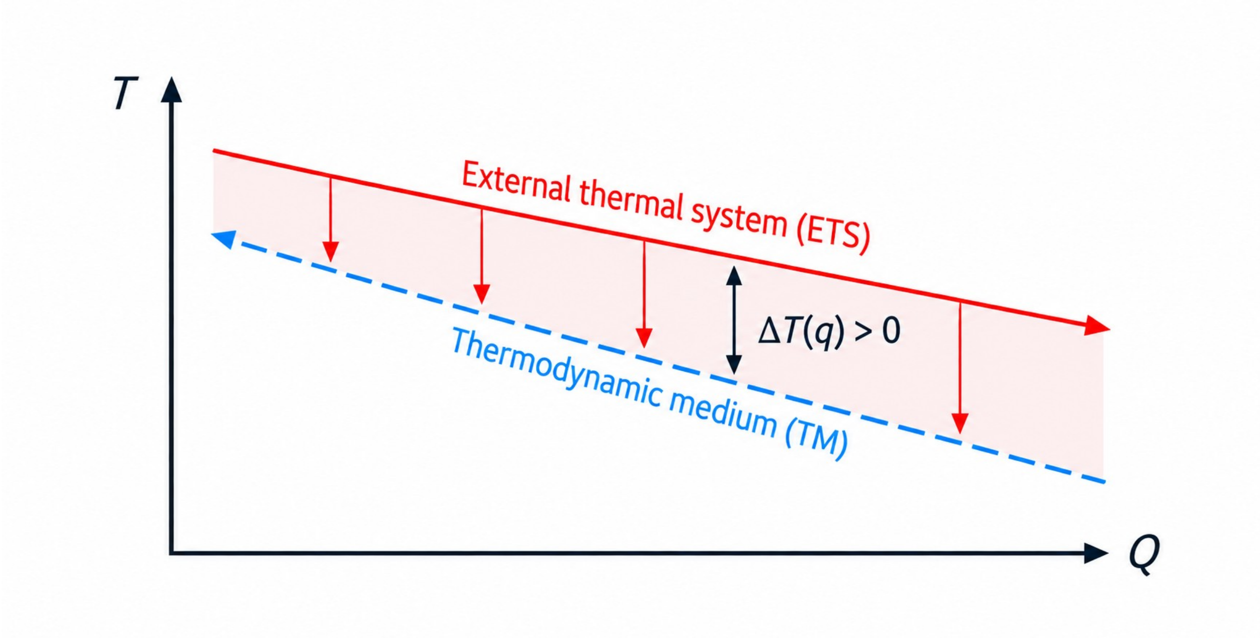
TECHNICAL METHODOLOGY REPORT

Thermal Matching in Non-Isothermal Thermodynamic Cycles

Making paired temperature trajectories visible, comparable and engineerable.



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Contents

Report at a Glance

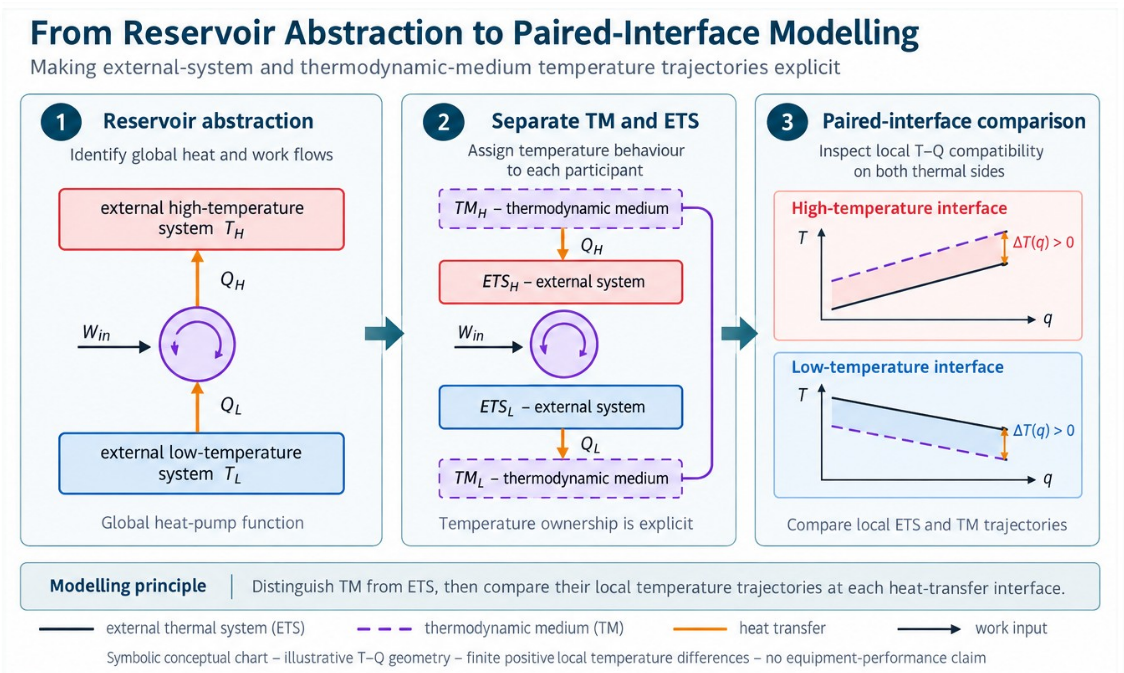
From Benchmark Framework to Thermal Matching

For engineers analysing heat engines and heat pumps, the proposed Clausius-based benchmark framework connects reversible-cycle reference conditions to explicit heat-transfer pairing. This report develops the associated Thermal Matching method: making paired temperature trajectories visible, comparable and engineerable.

The Formal Core is the foundational scientific research document of the CarnotX programme. It presents a new derivation and proposed conclusion for the Clausius Reversible-Cycle Benchmark Framework, with thermodynamic substantiation, worked calculations and applications. The Formal Core proposes the Clausius reversible-cycle equality as the generic parent benchmark relation. Carnot is its exact two-isothermal specialization, not the generic parent benchmark; the familiar Carnot efficiency and COP relations remain exact in that domain.

Thermal Matching engineers actual heat-transfer configurations. Their corresponding reversible benchmarks are constructed within the complete proposed framework using the equality, First-Law closure, process relations and stated boundary, reversibility, admissibility and selection conditions.

At the high- and low-temperature interfaces, the method distinguishes the thermodynamic medium (TM) from the external thermal system (ETS), assigns donor and receiver roles, and plots both boundary-temperature trajectories against a common transferred-heat coordinate. This pairing builds on their established physical distinction and reveals local ΔT(q). ETS includes ideal reservoirs and finite-capacity streams.



Conceptual overview. The chart provides a reader-facing map from reservoir abstraction to explicit TM/ETS identification and local paired-interface comparison; Chapters 3 and 4 develop the architectures explicitly. The Formal Core separately provides the corresponding reversible-cycle construction and benchmark-relative lost/excess-work framework.

Source: author's own elaboration.

The Thermal Matching Objective

Thermal Matching seeks to transfer a greater fraction, or where feasible the complete heat duty, across smaller admissible local ΔT(q). It changes temperature levels, trajectory shapes or heat-duty distribution. Improvement caused solely by increased conductance UA is classified separately as conductance enhancement.

Under a thermodynamically qualified like-for-like comparison, smaller relevant local differences move the affected heat-transfer process toward its reversible limit and reduce its attributable irreversibility. Function, duty, boundaries and permitted operating changes must be stated.

Isothermal behaviour and reversibility are independent classifications. Two constant temperature trajectories separated by a positive gap represent isothermal, irreversible heat transfer. The reversible heat-transfer limit requires pointwise coincidence of the paired boundary temperatures throughout the interaction; one zero terminal pinch is insufficient.

Three Simple Cases Used in This Report

The low-side heat-pump cases show how the same minimum approach can conceal different process-wide trajectories, distinguishing external-system changes from medium-cycle changes. Chapter 5 develops the cases; Chapter 6 adds dual-glide matching at both interfaces while retaining finite local differences.

Table 1. Three Simple Cases Used in This Report

Case	External system	Thermodynamic medium	What the paired view reveals
Fully isothermal	Constant at 10 °C	Constant at 5 °C	Both trajectories are horizontal, but remain 5 K apart
External-source glide	20 → 10 °C	Constant at 5 °C	External-system temperature changes while TM remains isothermal
TM glide	Constant at 10 °C	-5 → 5 °C	TM temperature changes while the external system remains isothermal

The numerical values and linear trajectories are illustrative, not equipment simulations or optimized designs. The cases retain a 10 K high-side approach and a 5 K minimum low-side approach.

Scope and Scientific Status

Two assessment levels. Paired T–Q trajectories and local ΔT(q) do not alone determine COP, a universal scalar irreversibility measure or an interface lost-work value. Cycle-level lost/excess work requires a complete actual-to-reversible comparison under declared functions, boundaries, useful duties and comparison conditions. Chapter 7 distinguishes these levels; §1.5 retains the limitations.

The method transfers to heat engines and refrigerators; Chapter 8 gives a bounded Brayton-type application. Established thermodynamics and temperature-profile matching underpin the analysis. The integrated methodology and Clausius-parent framework remain programme-specific proposals; cases are illustrative and performance implications require validation. This companion can be read independently of the Formal Core.

01 | Purpose, Scope and Scientific Status

This chapter defines why the report exists, what problem it addresses, and what it deliberately does not attempt to do. It positions the document as a bounded technical companion for paired-interface modelling rather than as a replacement for the Formal Core. It also introduces the cross-cycle transferability of Q–T thermodynamic engineering and separates established physical context from proposed method, illustrative case construction and broader programme implications.

1.1 Primary purpose

The primary purpose is to make the physical heat-transfer pairing explicit where Thermal Matching can be assessed. At each interface, the reader must be able to identify the heat donor, the heat receiver, the temperature trajectory of the thermodynamic medium, the temperature trajectory of the external thermal system and the direction of the heat quantity. The paired-interface view is especially consequential when at least one participant changes temperature through the heat-transfer process, because trajectory-shape mismatch then becomes an additional design variable. The fully isothermal case is retained as a baseline and may still admit Thermal Matching through deliberate temperature-level alignment that is distinct from conductance enhancement.

1.2 Scope

The worked demonstration is a closed heat-pump cycle with one net low-temperature heat-receiving branch, one net high-temperature heat-supplying branch and net work input. Only linear $T(Q)$ behaviour is considered for the non-isothermal illustrative cases. This corresponds to a constant effective dT/dQ over the stated interval, as can approximate a single-phase water stream with approximately constant heat-capacity rate under a bounded model. The fully isothermal case establishes the reference condition in which no profile-shape mismatch exists; Canonical Cases II and III then introduce one non-isothermal participant at the low-side interface. Chapter 3 uses an isothermal heat-engine counterpart to establish the paired-interface principle; Chapter 4 transfers the same visual grammar to the heat pump, which is the detailed demonstration used throughout the remainder of the report.

1.3 Cross-Cycle Transferability of Q–T Thermodynamic Engineering

In this report, Q–T thermodynamic engineering denotes analysis and design focused on heat quantities and their associated temperature trajectories within the externally coupled heat-transfer branches of a cycle. Within this bounded scope, heat engines, heat pumps and refrigerators share a common paired-interface grammar: heat-transfer direction, temperature ownership, donor–receiver relation, T(Q) trajectory shape, local $\Delta T(q)$, Thermal Matching and the reversible local limit. Cycle function changes the directions of heat and work and the applicable performance metric; component, mechanical, chemical, control and equipment engineering remain cycle- and machine-specific.

This creates direct cross-domain transferability. A water–water heat-pump interface with a sensible-water temperature glide and an approximately isothermal refrigerant branch, and the simplified

Brayton-type gas-turbine representation used later with an approximately isothermal external source or sink and a linearly varying thermodynamic-medium trajectory, share the same broad isothermal–linear paired-interface geometry. The gliding participant and energy-flow directions differ, but the local Q–T engineering problem is closely related; "isothermal–linear paired-interface class" is a report-specific descriptive label, not established nomenclature.

For a heat-pump specialist, the Brayton-type example therefore exposes a directly relevant Q–T trajectory-engineering problem. Under exact reversible reversal, both the medium cycle and the reversible external-boundary temperature trajectories may be retained while heat and work directions reverse; with finite-temperature-difference heat transfer, that full symmetry is lost. Section 3.3 develops this distinction explicitly.

1.4 Epistemic status

This companion combines established thermodynamics with Formal-Core interpretation, report-specific visual conventions and illustrative examples. The table below therefore separates what is established, what is proposed or interpreted within the programme, and what remains illustrative or unvalidated.

Table 2. *Epistemic Status*

Layer	Status in this report
Established science / engineering context	First Law; heat transfer through finite temperature difference is irreversible; local equality is required for reversible heat transfer; COP definitions. T–Q temperature-profile analysis and temperature-glide matching are established in specialist heat-exchanger and zeotropic heat-pump literature.
Formal Core interpretation	Formal Core interpretation: Carnot is treated as the exact two-isothermal specialization, while the operationalized Clausius reversible-cycle equality is proposed as the parent framework for constructing corresponding reversible cycles across admissible non-isothermal configurations. This companion uses that positioning but does not reproduce the formal construction or lost-work method.
Proposed operational method	Progressive reservoir-only, A/A and B/B visual grammar, together with a Thermal Matching method aligned with the Formal Core: improved local alignment may involve trajectory shape, temperature level, heat-duty distribution, or combinations of these, and is distinct from conductance enhancement. The A/A and B/B codes are report-specific standardised reporting conventions. "Isothermal–linear paired-interface class" is also report-specific descriptive terminology, not established nomenclature.
Illustrative calculation	The selected 60/50 °C high side and 20/10/5/-5 °C low-side temperatures.
Not established here	Technical realizability, equipment performance, optimized refrigerant composition and independently validated gains.

1.5 Reporting Scope and Limitations

What the report does

Within that deliberately bounded scope, the report does the following: The report does not claim priority for temperature-profile matching itself.

- Defines the thermodynamic medium and external thermal system as separate physical participants and standardises energy-flow direction, temperature ownership, T–Q orientation, colours, line styles and minimum model annotations.

- Documents three canonical illustrative heat-pump cases and a dual-glide engineering synthesis under explicitly stated comparison assumptions, without identifying the improved configuration as the thermodynamic optimum or reversible limit.
- Uses the heat-engine directional counterpart as the conceptual bridge, makes the cross-cycle transferability of Q–T interface engineering explicit, applies the same visual grammar to heat-pump architectures, and adds a concise Brayton-type gas-turbine recognition case.
- Separates conductance enhancement from Thermal Matching and separates interface-level Thermal-Matching diagnostics from cycle-level work-consequence quantification, which remains part of the Formal Core method.
- Provides a reusable paired-interface protocol for isothermal, piecewise-isothermal, continuously non-isothermal and mixed cases under a declared comparison basis.

What the report does not do

The report deliberately stops short of a complete equipment-design or cycle-design method. The following limitations remain explicit:

- It does not prove that the compressor, expansion device, flow arrangement or refrigerant state path is reversible or technically realizable.
- It does not infer complete-cycle reversibility from zero temperature difference at the two external heat-transfer interfaces.
- It does not treat one zero terminal pinch as proof of process-wide reversible heat transfer.
- It does not assume that increasing U or the heat-transfer conductance UA changes temperature profiles unless the comparison class permits the operating state, flow, area, duty or terminal temperatures to change.
- It does not use the local finite temperature difference of one heat-transfer process as a standalone scalar measure of the thermodynamic magnitude of irreversibility. Interface-level assessment characterises where and how thermal mismatch occurs; cycle-level work consequence is quantified separately through the complete-cycle actual-to-reversible comparison.
- It does not treat the Brayton-type gas-turbine example as a complete combustor or open-cycle engine model. The isothermal source/sink representation is an illustrative air-standard abstraction; real combustion heat release, gas properties, fuel addition, cooling flows and exhaust processes are more complex.

02 | System Boundary, Entities and Directions

This chapter fixes the basic modelling language used throughout the report. It defines the cyclic thermodynamic medium, the external thermal systems, the heat-flow directions and the naming conventions needed for unambiguous paired-interface diagrams. By making those entities and directions explicit at the outset, later figures can be read without ambiguity about temperature ownership or transfer direction.

2.1 Investigated cycle

The investigated cyclic system is the thermodynamic medium. The labels TM_H and TM_L identify its high- and low-temperature heat-transfer stages; they do not denote two separate media. The external high-temperature sink and low-temperature source remain outside that cyclic-system boundary.

2.2 External thermal system as umbrella term

External thermal system (ETS) is used as the umbrella term. An ideal isothermal thermal reservoir is one special case. A flowing water stream whose boundary temperature changes through the heat exchanger is more accurately described as a non-isothermal external thermal stream or finite-capacity thermal source. Canonical Case II is therefore referred to as External-Source Glide.

2.3 Energy directions

The energy-flow directions are fixed first so that donor-receiver roles and all subsequent temperature-difference definitions remain unambiguous.

$$\mathrm{ETS}_L \xrightarrow{Q_L} \mathrm{TM}_L \rightarrow \mathrm{cycle} \rightarrow \mathrm{TM}_H \xrightarrow{Q_H} \mathrm{ETS}_H$$

At the low-temperature interface, the external thermal source is the donor and the thermodynamic medium is the receiver. At the high-temperature interface, the thermodynamic medium is the donor and the external thermal sink is the receiver. Net work enters the cycle.

$$Q_H = Q_L + W_{\mathrm{in}} \tag{1}$$

BOUNDARY RULE
Every temperature shown in a heat-transfer statement belongs to one declared physical side of the interface. A bulk stream temperature is used only when it adequately represents the boundary temperature for the stated model.

2.4 Local heat coordinate

Each interface uses its own cumulative positive heat magnitude Q_j and normalized coordinate $q_j = Q_j / Q_{j,tot}$. Both paired temperature trajectories on one interface are plotted against that same coordinate. High- and low-side plots may use the same normalized horizontal scale while representing different total heat quantities; Q_H and Q_L must not be visually or algebraically conflated.

$$q_j = \frac{Q_j}{Q_{j,\mathrm{tot}}}, \qquad 0 \le q_j \le 1 \tag{2}$$

2.5 Donor-receiver temperature difference

At each interface, the local temperature difference is defined from heat donor to heat receiver.

$$\Delta T_j(q) = T_{\mathrm{donor},j}(q) - T_{\mathrm{receiver},j}(q) \ge 0 \tag{3}$$

Equation (3) is used as a direct local diagnostic. The report does not collapse a varying gap into a universal scalar Thermal Matching index. Minimum, maximum and terminal approaches may be stated, but none alone represents the complete trajectory.

2.6 Terminology discipline

Because the same temperature information can support different process- and cycle-level statements, the wording used throughout the report is deliberately controlled. The conventions below keep local heat-transfer irreversibility, complete-cycle irreversibility and qualitative interpretation distinct.

- Process statement: 'the heat-transfer process is thermodynamically irreversible because a finite local boundary-temperature difference is present.'
- Cycle statement: 'the complete cycle is irreversible' only after all relevant thermal and non-thermal mechanisms are included.
- Zero statement: identify the exact quantity, interface, interval and assumptions; do not infer absence of cycle irreversibility from cyclic state restoration.
- Energy-degradation language may be used as qualitative interpretation, but a quantitative value must name its defined method, boundary and comparison class.

Next — paired-interface principle. With the cycle boundary, physical participants, heat directions and temperature ownership now fixed, Chapter 3 turns these definitions into an explicit paired-interface visual architecture using the heat-engine counterpart.

03 | Heat-Engine Counterpart and Paired-Interface Principle

> *This chapter introduces the paired-interface principle through the simpler directional counterpart of the heat engine. It shows, in a controlled progression, what a conventional reservoir-only representation reveals and what remains implicit until the thermodynamic-medium and external-system trajectories are separated. It also distinguishes exact reversible cycle reversal from finite-temperature-difference reversal, clarifying where heat-engine/heat-pump Q–T symmetry holds and where it breaks before the same visual grammar is transferred to the heat-pump cases.*

3.1 Heat-engine counterpart progression

The progression below uses three heat-engine representations to show, step by step, what remains implicit in a reservoir-only sketch, what becomes explicit in the paired-interface view, and what changes when finite temperature differences are introduced.

Table 3. *Heat-Engine Counterpart Progression*

Representation	Visible information and diagnostic consequence
Heat engine - compact traditional bridge	Begins with the familiar reservoir-cycle-reservoir energy-flow sketch. The medium-side branches remain implicit on the left, while paired T–Q plots make their boundary temperatures visible.
Heat engine - explicit A/A, zero ΔT	Expands TM_H and TM_L as separate stages of one cyclic medium. Coincident horizontal trajectories establish the reversible heat-transfer limit at both interfaces.
Heat engine - finite ΔT; A/A = B/B	Places each donor above its receiver at both interfaces: $TR_H \rightarrow TM_H$ and $TM_L \rightarrow TR_L$. Because both heat flows are downward, the energy-flow and paired-interface object orders coincide in this special isothermal case. The 10 K and 5 K gaps make both transfers irreversible.

3.2 Visual semantics and transferability

The figures use one consistent visual grammar so that colour, line style, heat-flow direction and trajectory geometry carry the same meaning throughout the report. The following rules define that reading.

- Coral family: high-temperature interaction; blue family: low-temperature interaction; magenta: cycle device or cyclic-medium route.
- Heat-engine reading: $TR_H \rightarrow TM_H$ and $TM_L \rightarrow TR_L$; W_{out} leaves the cycle. In the finite-ΔT isothermal figure, donor-above-receiver ordering makes A/A and B/B coincide as a physical object arrangement. This is a special-case coincidence, not a general equivalence.
- Transfer to Chapter 4: for the heat pump, $TR_L \rightarrow TM_L$ and $TM_H \rightarrow TR_H$ while W_{in} enters the cycle. The reversed heat directions make the difference between A/A energy-flow organisation and B/B paired-interface organisation especially visible.

- Horizontal T–Q trajectories denote isothermal behaviour. Coincident trajectories represent the reversible heat-transfer limit; separated trajectories represent finite-temperature-difference heat transfer. Complete-cycle reversibility additionally requires every remaining cycle process to be reversible.

> **PROPOSED-METHOD STATUS** The reservoir-only sketch is a conventional pedagogical abstraction. A/A and B/B are report-specific architecture codes: A/A denotes an explicit energy-flow organisation and B/B an explicit paired-interface organisation. The equality A/A = B/B stated for Figure 3 concerns only the object ordering of the displayed finite-ΔT isothermal heat-engine case; it is not a universal identity or established international nomenclature.

3.3 Reversible and Irreversible Heat-Engine/Heat-Pump Reversal

Cycle reversal provides a direct way to distinguish what is common to heat-engine and heat-pump Q–T engineering from what changes when finite temperature differences are retained. The argument below concerns the external heat-transfer branches and follows from established local heat-flow direction and reversible heat-transfer conditions; it does not assert equivalence of complete equipment architectures or of non-thermal irreversibilities, and no new thermodynamic law is claimed.

3.3.1 Reversible reversal — common cycle and boundary trajectories

At the reversible heat-transfer limit, the thermodynamic-medium and external-system boundary temperatures coincide pointwise at each heat-transfer interface:

$$T_{\mathrm{TM}}(\mathrm{q}) = \mathrm{T}_{\mathrm{ETS}}(\mathrm{q})$$

A reversible heat engine and its corresponding reversed heat-pump cycle can therefore be represented as the same thermodynamic cycle traversed in opposite directions. The thermodynamic states, process paths, reversible external-boundary temperature trajectories and absolute heat and work quantities can remain the same, while the directions of QH, QL and W reverse. Complete-cycle reversibility additionally requires every remaining process to satisfy its reversible-limit conditions.

3.3.2 Irreversible reversal — loss of simultaneous symmetry

With finite local temperature differences, a physically admissible heat-transfer process requires the local heat donor to be hotter than the local receiver. An irreversible heat engine and a corresponding heat-pump arrangement therefore cannot simultaneously retain both of the following under reversal:

- exactly the same thermodynamic-medium cycle traversed in the opposite direction; and
- exactly the same external thermal systems and temperature trajectories.

At least one side of the paired interface must change when the heat-flow direction is reversed while a finite driving temperature difference is retained.

3.3.3 Same medium cycle, different external systems

If the thermodynamic medium is constrained to traverse exactly the same cycle in the opposite direction, its heat-transfer temperatures and T–Q trajectories remain fixed while the heat-flow directions reverse. For the simplified isothermal high- and low-side ordering used here, the heat engine requires:

$$T_{\mathrm{H,ETS}} > \mathrm{T}_{\mathrm{H,TM}} \quad \text{and} \quad \mathrm{T}_{\mathrm{L,TM}} > \mathrm{T}_{\mathrm{L,ETS}}$$

whereas the reversed heat-pump heat-flow directions require:

$$T_{\mathrm{H,TM}} > \mathrm{T}_{\mathrm{H,ETS}} \quad \text{and} \quad \mathrm{T}_{\mathrm{L,ETS}} > \mathrm{T}_{\mathrm{L,TM}}$$

The same medium cycle can therefore not be paired with the same finite-ΔT external temperature levels or T–Q trajectories in both operating directions.

3.3.4 Same external systems, different medium cycles

If the external thermal systems are instead retained, the thermodynamic-medium temperatures must move to the opposite side of the external trajectories so that heat continues to flow from donor to receiver. In the simplified two-isothermal-level comparison, the heat-engine medium temperature span is smaller than the corresponding external-system span:

$$T_{\mathrm{H,TM}} - \mathrm{T}_{\mathrm{L,TM}} < \mathrm{T}_{\mathrm{H,ETS}} - \mathrm{T}_{\mathrm{L,ETS}}$$

whereas the heat-pump medium temperature lift must exceed the external-system temperature lift:

$$T_{\mathrm{H,TM}} - \mathrm{T}_{\mathrm{L,TM}} > \mathrm{T}_{\mathrm{H,ETS}} - \mathrm{T}_{\mathrm{L,ETS}}$$

The irreversible heat engine and heat pump therefore follow different thermodynamic-medium cycles when the same external systems are retained. These span inequalities describe the present simplified isothermal-level ordering and are not introduced as universal scalar criteria for arbitrary non-isothermal cycles.

3.3.5 Extension to non-isothermal trajectories

For non-isothermal cycles, the same principle applies pointwise over the complete transferred-heat coordinate. Not only the temperature levels but also the shapes, slopes and local separations of the paired T–Q trajectories may differ. A corresponding irreversible non-isothermal heat engine and heat pump may therefore belong to the same cycle family or share the same Q–T analysis grammar without being the same thermodynamic cycle with only the energy-flow directions reversed.

REVERSAL PRINCIPLE — Only in the reversible heat-transfer limit can both the thermodynamic-medium cycle and the external thermal-system temperature trajectories remain unchanged under exact cycle reversal. With finite-temperature-difference heat transfer, reversal requires either the external temperature trajectories to change when the medium cycle is retained, or the medium cycle to change when the external systems are retained.

3.4 Traditional Compact Heat-Engine Representation

The sequence begins from the familiar compact heat-engine sketch so that the information added by paired-interface modelling can be seen explicitly. Figure 1 therefore serves as the reference abstraction against which Figures 2 and 3 are read.

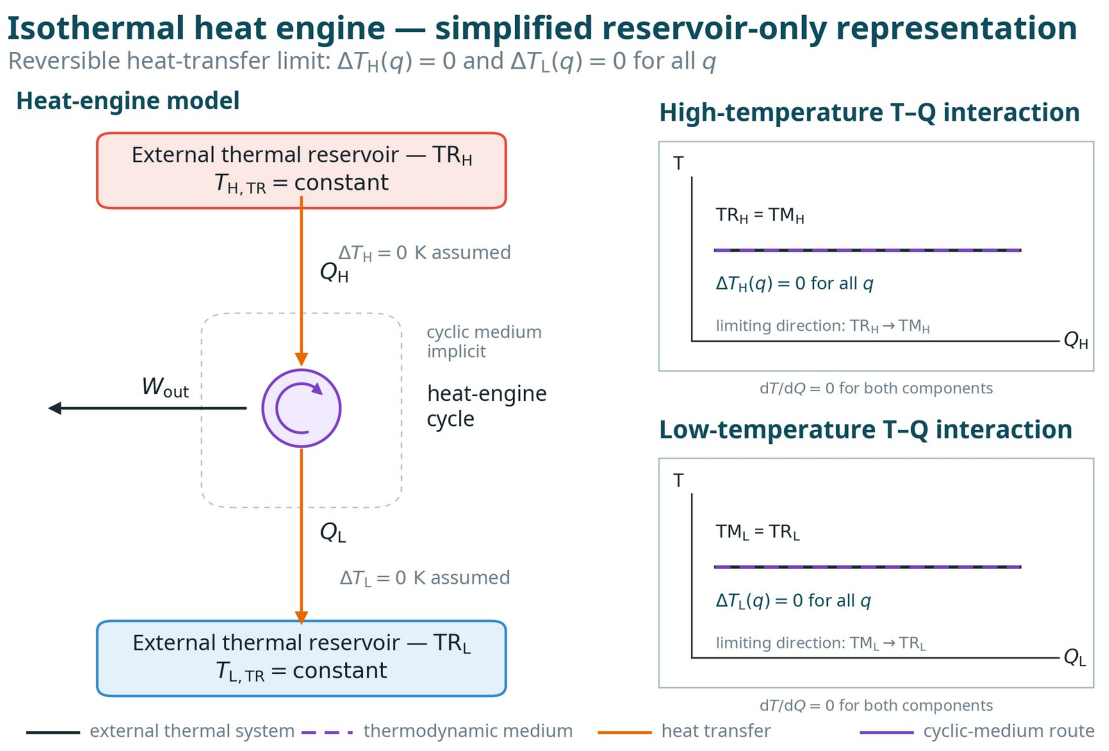


Figure 1. *Traditional compact reservoir-only representation of an isothermal heat engine, retaining the familiar reservoir-cycle-reservoir energy-flow organisation on the left and adding paired T–Q diagnostics on the right. Q_H flows from TR_H into the cycle, W_{out} leaves the cycle and Q_L is rejected to TR_L. Coincident TR/TM trajectories demonstrate $\Delta T_H = \Delta T_L = 0$ K. The A/A architecture is introduced only in Figure 2.*

Source: author's own elaboration.

TRADITIONAL STARTING POINT The left-hand sketch keeps TM_H and TM_L implicit within the cycle symbol, as in many classical heat-engine schematics. The paired T–Q plots nevertheless expose both medium-side temperatures and their limiting heat-flow directions. The cycle balance is $Q_H = W_{out} + Q_L$; reversible-limit heat transfer is established only because the paired temperatures coincide pointwise.

3.5 Explicit A/A Heat-Engine Reversible-Limit Architecture

The next step retains the same thermal case but makes the high- and low-temperature thermodynamic-medium stages explicit. Figure 2 shows what changes when temperature ownership is disclosed without changing the cycle itself.

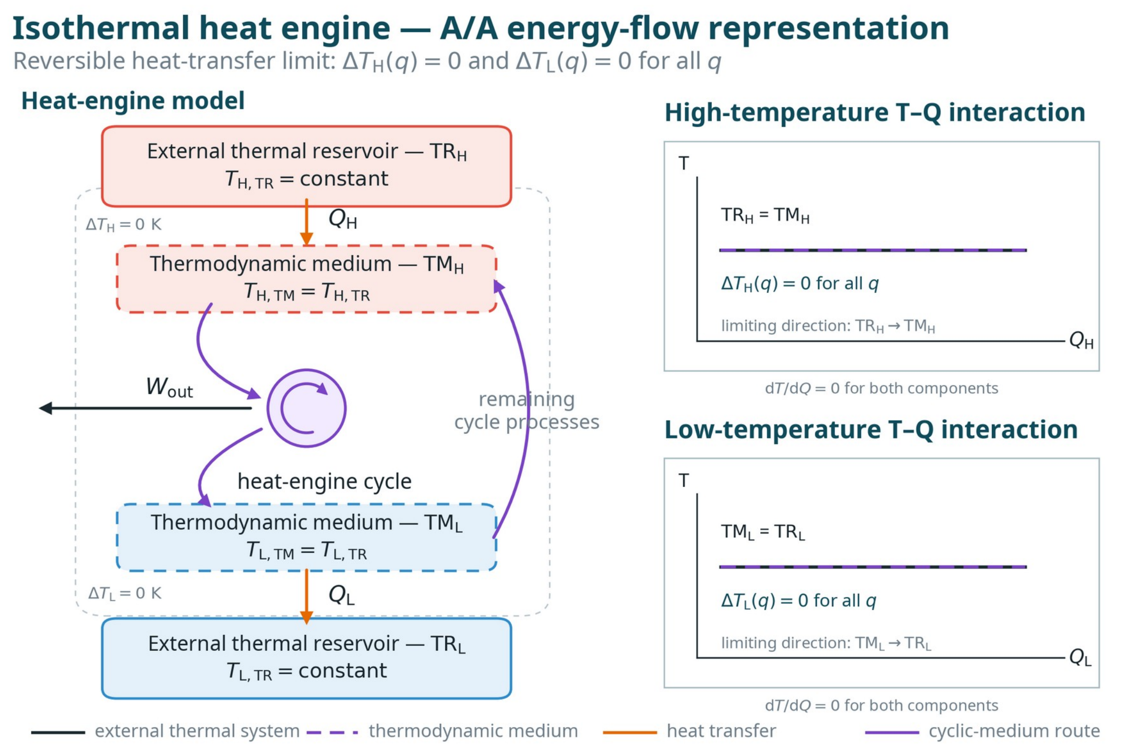


Figure 2. *Fully explicit A/A energy-flow representation of the same zero-ΔT isothermal heat-engine case as Figure 1. Separate* TM_H *and* TM_L *objects are placed on one cyclic-medium route. This makes temperature ownership and the connection through the remaining cycle processes explicit without changing temperatures, heat directions or cycle function.*

Source: author's own elaboration.

SAME CASE, GREATER ARCHITECTURAL DISCLOSURE Figures 1 and 2 represent the same heat engine at the reversible heat-transfer limit. Figure 2 expands the previously implicit high- and low-temperature branches of the thermodynamic medium. TM_H and TM_L are therefore two heat-transfer stages of one cyclic medium, not two separate media, and complete-cycle reversibility still requires all remaining cycle processes to be reversible.

3.6 Coincident A/A and B/B Finite- Heat-Engine Architecture

The final heat-engine step changes only the thermal condition from zero to finite approach temperatures. Figure 3 then shows why A/A and B/B happen to coincide in object order for this special isothermal heat-engine arrangement.

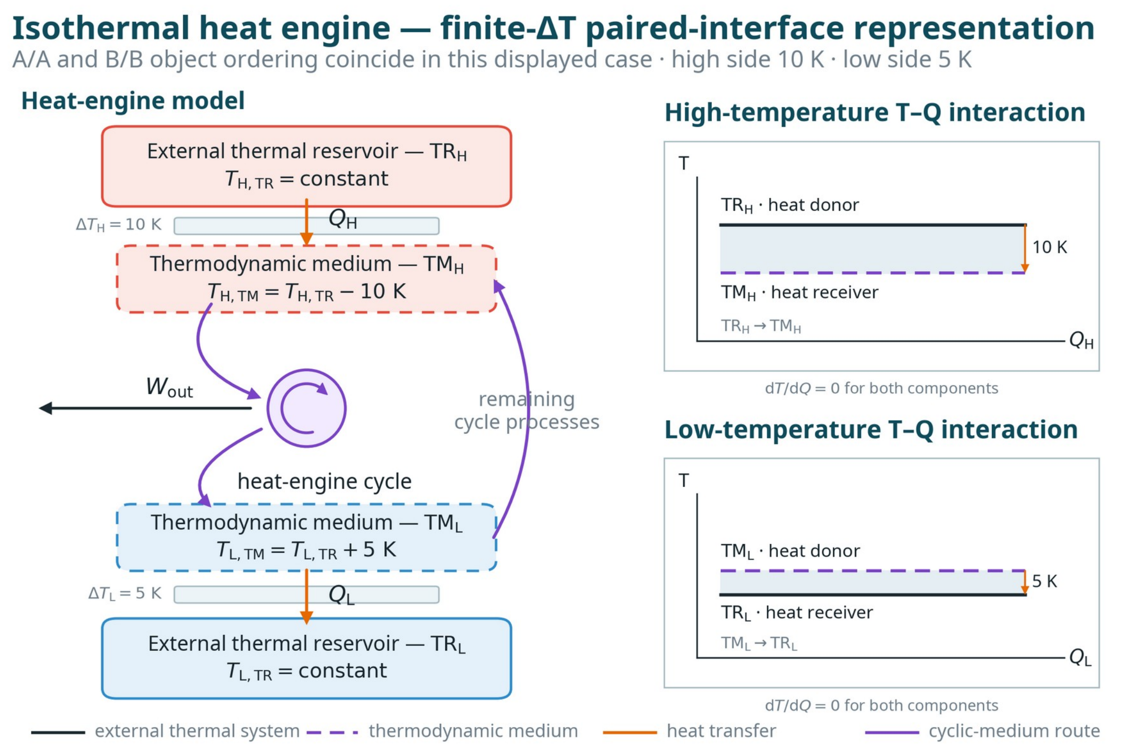


Figure 3. *Isothermal heat engine with a 10 K high-side and 5 K low-side temperature difference.* TR_H *is the high-side donor and* TM_H *the receiver;* TM_L *is the low-side donor and* TR_L *the receiver. Because donor-to-receiver heat transfer is downward at both interfaces, the explicit A/A energy-flow order and B/B paired-interface order coincide in this special case. The separated horizontal T–Q trajectories have no profile-shape or slope mismatch, yet the finite temperature gaps make both transfers irreversible.*

Source: author's own elaboration.

A/A = B/B IN THIS DISPLAYED CASE The equality describes an architectural coincidence, not a universal identity. With TR_H above TM_H and TM_L above TR_L, both A/A and B/B place the same four objects in the same vertical order while preserving $Q_H = W_{out} + Q_L$. Compared with Figures 1 and 2, Figure 3 also changes the thermal condition from zero to finite gaps. The two interface transfers are irreversible, while complete-cycle reversibility or irreversibility still requires assessment of all remaining cycle processes.

CHAPTER 3 — KEY CONCLUSION

The heat-engine sequence establishes the paired-interface principle by making the thermodynamic-medium heat-transfer stages explicit alongside the external thermal systems. This separates temperature ownership, heat-transfer direction and local temperature differences that remain hidden in a reservoir-only sketch. It also reinforces that isothermal behaviour and reversibility are independent classifications.

NEXT — HEAT-PUMP ARCHITECTURE

Chapter 4 applies the same physical grammar to the heat pump, where the reversed heat directions make the A/A energy-flow and B/B paired-interface organisations more visibly distinct and establish the reference architecture for the canonical cases.

04 | Heat-Pump Paired-Interface Architecture

This chapter transfers the paired-interface grammar from the heat-engine bridge case to the heat pump. It reconstructs the heat-pump architecture step by step, starting from the traditional reservoir abstraction and moving to explicit A/A and B/B representations. The result is a stable visual and thermodynamic basis for the canonical comparisons developed in the next chapter.

4.1 Traditional Reservoir-Only Heat-Pump Representation

The heat-pump sequence starts again from the conventional reservoir-only abstraction. Figure 4 establishes the baseline before the thermodynamic-medium branches are made explicit.

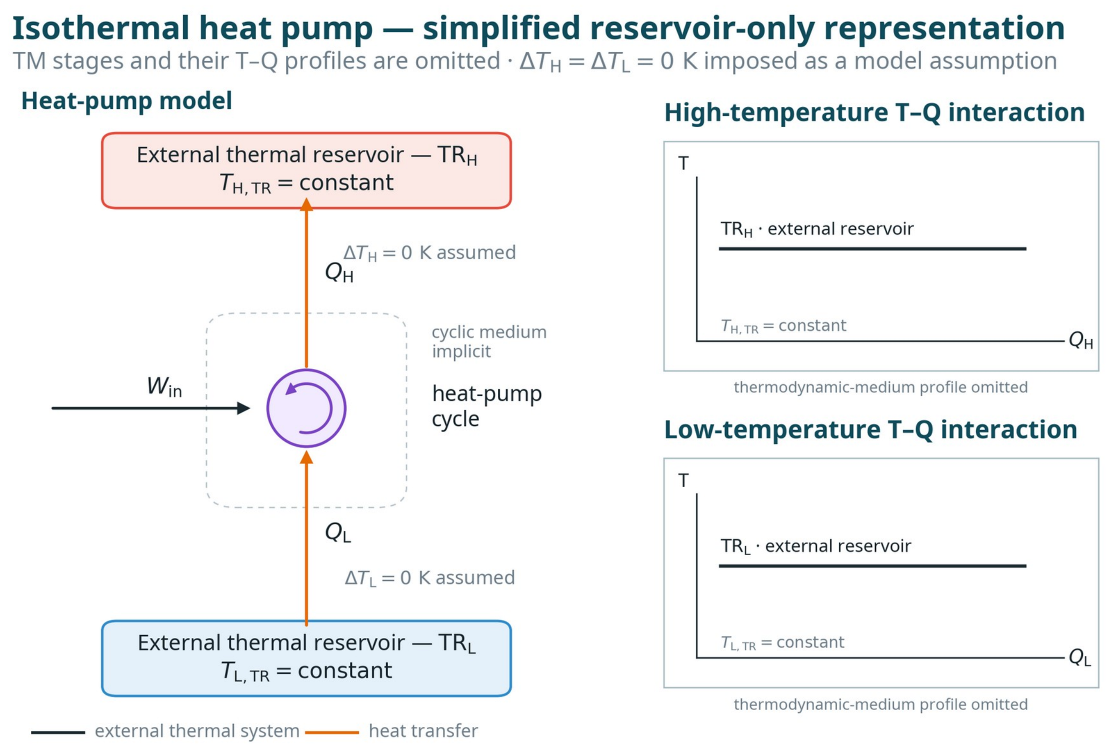


Figure 4. *Traditional simplified reservoir-only representation of an isothermal heat pump. The external reservoirs, heat quantities and work input are visible, while the thermodynamic-medium branches and their T–Q trajectories are omitted. The stated $\Delta T_H = \Delta T_L = 0$ K condition is imposed as a model assumption and is not demonstrated by the reservoir-only plots.*

Source: author's own elaboration.

INTERPRETATION

This compact abstraction identifies the heat-pump function and energy directions. Because the medium-side temperatures are implicit, it cannot independently diagnose the local thermal driving differences. It may represent a reversible heat-transfer limit only when equality of the omitted medium and reservoir temperatures is separately declared.

4.2 A/A Heat-Pump Reversible-Limit Architecture

The first explicit heat-pump architecture is the reversible local heat-transfer limit. Figure 5 shows how separate thermodynamic-medium and external-system objects coincide pointwise at both interfaces.

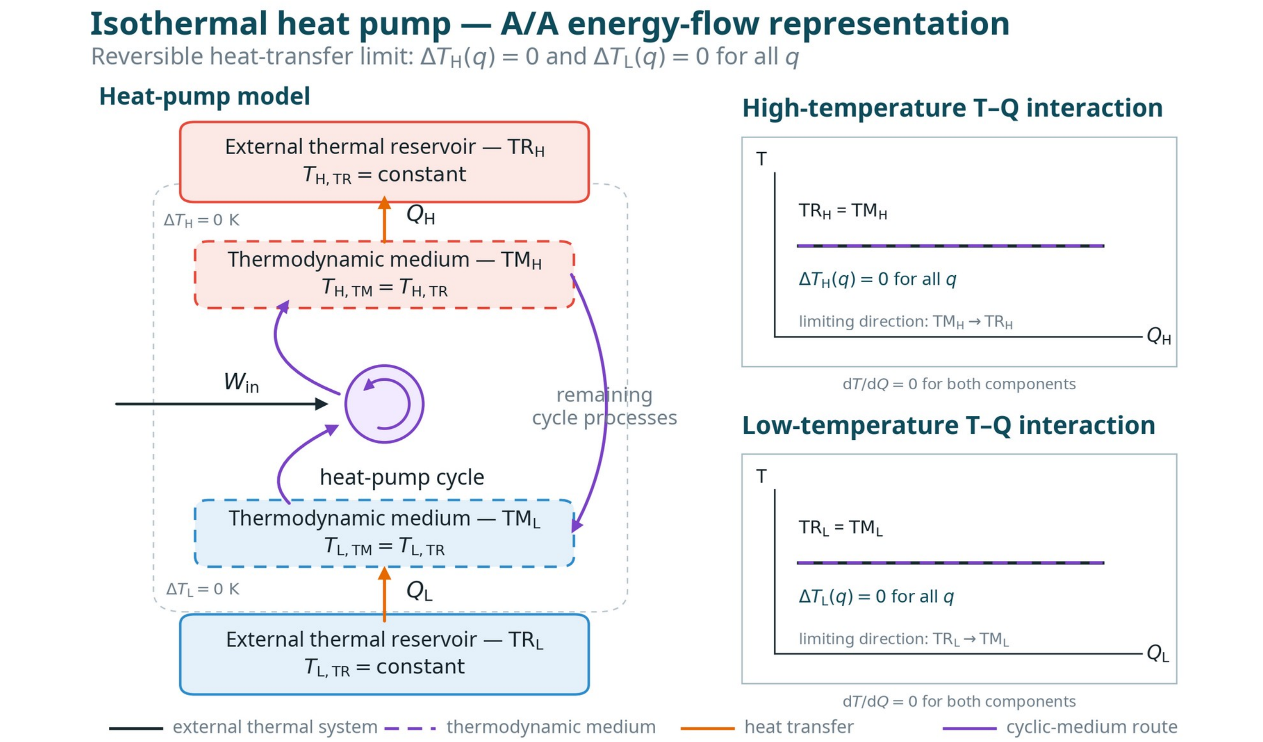


Figure 5. *A/A energy-flow architecture for the fully isothermal heat pump at the reversible heat-transfer limit. The thermodynamic medium and external reservoir are separate objects at both thermal sides; their coincident horizontal T–Q trajectories explicitly establish* $\Delta T_H = \Delta T_L = 0$ K.

Source: author's own elaboration.

REVERSIBILITY BOUNDARY

Pointwise temperature equality establishes reversible-limit heat transfer at the two displayed interfaces. It does not by itself establish that the complete heat-pump cycle is reversible; all remaining internal processes must also satisfy their reversible-limit conditions.

4.3 A/A Heat-Pump Finite-ΔT Architecture

Keeping the same explicit A/A architecture, Figure 6 introduces finite approach temperatures while all four trajectories remain isothermal. This isolates finite-temperature-difference irreversibility without adding trajectory-shape mismatch.

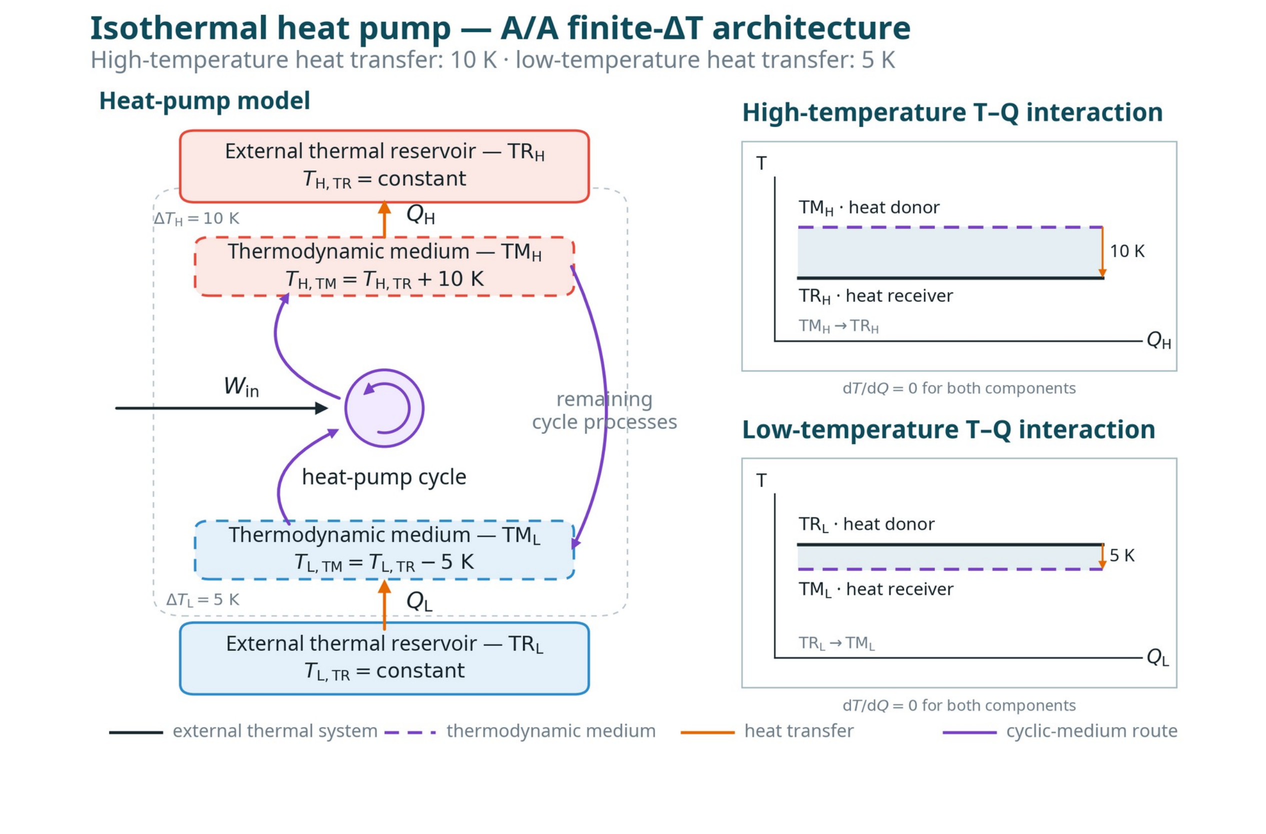


Figure 6. *A/A energy-flow architecture for the same fully isothermal heat-pump function with a 10 K high-side and 5 K low-side temperature difference. All four boundary-temperature trajectories remain horizontal, but finite separation makes heat transfer irreversible at both interfaces.*

Source: author's own elaboration.

ISOTHERMAL IS NOT REVERSIBLE
The thermodynamic medium and reservoirs are isothermal during their respective heat interactions, yet their unequal temperatures create finite-temperature-difference irreversibility. Both paired profiles have zero slope, so no profile-shape mismatch is present. Their finite separation can nevertheless be reduced either by conductance enhancement or by a direction-neutral change to one or both temperature levels that is not attributable solely to increasing UA; these interventions must not be conflated.

4.4 B/B Heat-Pump Finite-ΔT Architecture

Figure 7 reorganises the same finite-gap heat-pump case by pairing each donor directly with its receiver. The change is representational rather than thermodynamic.

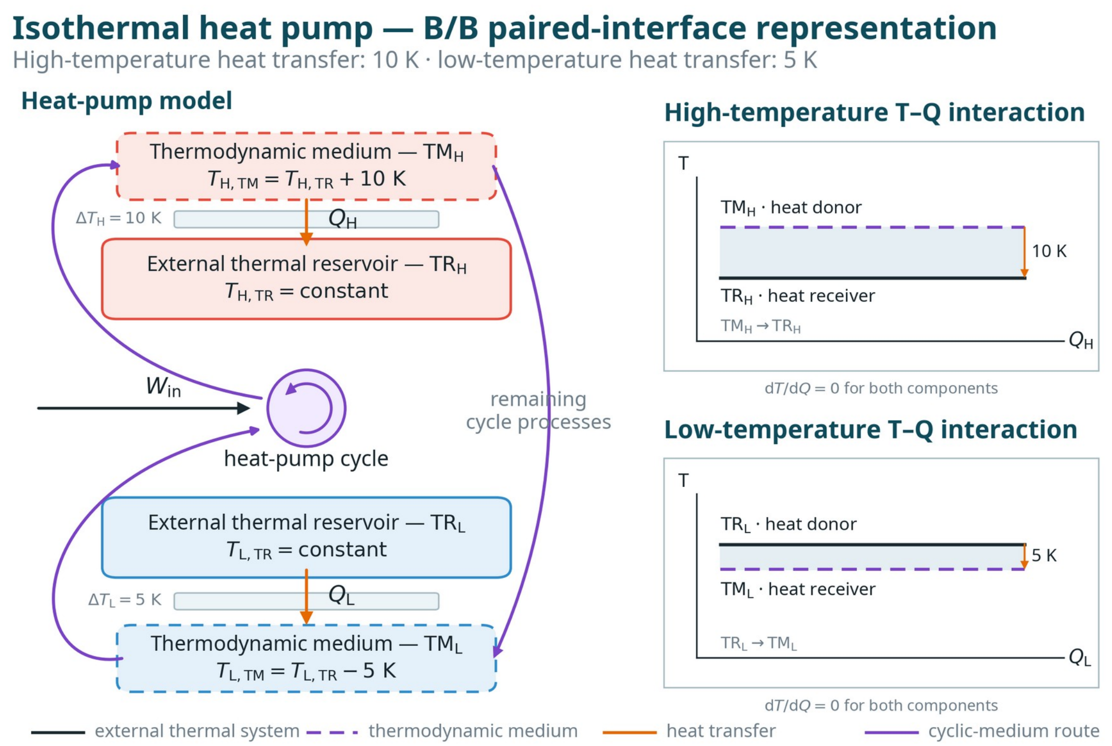


Figure 7. *B/B paired-interface architecture applied to the same fully isothermal finite-ΔT heat-pump case as Figure 6. Donor and receiver are positioned directly across each interface and their horizontal T–Q trajectories expose the 10 K high-side and 5 K low-side separations.*

Source: author's own elaboration.

> **A/A-B/B READING RULE** Figures 6 and 7 do not represent different heat-pump cycles. They preserve the same temperatures, heat directions and cycle function. B/B changes the visual organisation by pairing each heat donor and receiver locally, making temperature ownership, interface direction and the finite gap easier to inspect.

Next — thermodynamic conditions. The heat-pump architecture now makes both sides of each thermal interface explicit without changing the cycle function. Section 5.1 shifts from representation to the thermodynamic classification of temperature behaviour, local heat-transfer reversibility and complete-cycle reversibility.

05 | Thermodynamic Classification and Canonical Cases

This chapter states the governing thermodynamic distinctions used for paired-interface interpretation and then applies them to the three canonical heat-pump cases. It distinguishes isothermal from non-isothermal behaviour and keeps that classification separate from the independent question of reversibility or irreversibility. The chapter closes by comparing the cases directly so that the physical meaning of each configuration becomes explicit.

5.1 Governing Thermodynamic Conditions

5.1.1 Interaction classification

An interaction is isothermal when both relevant boundary temperatures remain constant throughout that interaction. A finite constant difference does not change that classification. An interaction is non-isothermal when at least one relevant boundary-temperature trajectory varies through the heat-transfer process.

Isothermal versus non-isothermal and reversible versus irreversible are independent classifications. The first describes temperature behaviour during heat transfer; the second describes thermodynamic reversibility. An isothermal heat-transfer interaction may be irreversible when a finite boundary-temperature difference is present, while a non-isothermal interaction may satisfy the reversible heat-transfer limit when the paired boundary-temperature trajectories coincide pointwise throughout the interaction.

5.1.2 Reversible local thermal limit

The reversible local heat-transfer limit is a pointwise boundary condition over the complete heat-transfer coordinate, not merely an endpoint condition.

$$T_{\text{donor},j}(q) = T_{\text{receiver},j}(q) \text{ for every } q \in [0,1] \tag{4}$$

Pointwise coincidence is the thermal-interface condition. Equal endpoint temperatures, one zero pinch or equal average temperatures do not establish Equation (4) over the complete interaction.

5.1.3 Linear trajectory model

The illustrative non-isothermal cases use a linear T–Q trajectory as the simplest controlled representation of temperature change with transferred heat.

$$T(q) = T_{\text{in}} + (T_{\text{out}} - T_{\text{in}})\,q \tag{5}$$

$$\frac{dT}{dQ} = \frac{T_{\text{out}} - T_{\text{in}}}{Q_{\text{tot}}} = \text{constant} \tag{6}$$

The linear model is an idealized representation. A real water or refrigerant trajectory can depart from linearity because heat capacity, phase equilibrium, pressure, composition, heat loss or boundary conditions vary.

The horizontal coordinate in a *T–Q* representation is transferred heat, not physical distance along a heat exchanger. A linear *T(Q)* trajectory therefore describes a linear thermodynamic temperature response to transferred heat; it does not imply a linear spatial *T(x)* profile. Section 5.3.5 illustrates this distinction for the external-source-glide case.

5.1.4 Necessary but insufficient thermal condition

If Equation (4) holds on both heat-transfer branches, those two modelled heat-transfer processes are in their reversible limit. That result does not by itself prove complete-cycle reversibility. Friction, pressure losses, non-reversible compression or expansion, throttling, dissipative mixing, electrical resistance and other mechanisms may remain.

5.1.5 Logically distinct statements

These distinctions separate temperature behaviour, local heat-transfer reversibility and complete-cycle conclusions.

Table 4. *Logically Distinct Thermodynamic Statements*

Statement	What it establishes	What it does not establish
The interaction is isothermal or non-isothermal	Temperature behaviour during the heat-transfer interaction.	Reversibility or irreversibility.
Both paired trajectories are constant	The interaction is isothermal.	Reversible heat transfer.
Paired trajectories coincide pointwise	Reversible-limit heat transfer for that interface.	Complete-cycle reversibility.
A single terminal gap is zero	A local pinch at that endpoint.	Trajectory coincidence elsewhere.
First-Law balance closes	Energy accounting for the selected cycle boundary.	Admissibility, realizability or reversibility.

5.1.6 Profile mismatch in the canonical linear cases

For the present three-case set, the profile-shape comparison is deliberately simple. Two horizontal profiles have no slope mismatch, although a finite offset may remain. A horizontal profile paired with a nonzero linear slope has a structural shape mismatch. Chapter 6 defines the broader Thermal-Matching intervention space.

KEY CONCLUSION — Isothermal/non-isothermal and reversible/irreversible are independent classifications. Reversible local heat transfer requires pointwise equality of the paired boundary temperatures; complete-cycle reversibility also requires all remaining cycle processes to be reversible. Next — canonical cases. Sections 5.2–5.4 apply these conditions to the three heat-pump cases.

5.2 Canonical Case I - Fully Isothermal Heat Pump

> *All four boundary-temperature behaviours are constant; both interfaces retain a finite approach.*

5.2.1 Model definition

Canonical Case I defines the fully isothermal reference configuration used for the subsequent comparisons.

$$T_{H,\mathrm{TM}} = 60\,°\mathrm{C}, \qquad T_{H,\mathrm{ETS}} = 50\,°\mathrm{C}, \qquad \Delta T_H = 10\,\mathrm{K} \tag{7}$$

$$T_{L,\mathrm{ETS}} = 10\,°\mathrm{C}, \qquad T_{L,\mathrm{TM}} = 5\,°\mathrm{C}, \qquad \Delta T_L = 5\,\mathrm{K} \tag{8}$$

At the high side, the thermodynamic medium rejects Q_H to the external sink. At the low side, the external source supplies Q_L to the thermodynamic medium. Each donor and receiver remains at a constant boundary temperature throughout its respective heat-transfer process.

5.2.2 Classification

The case is classified below by cycle function, interaction structure, local heat-transfer reversibility, profile-shape mismatch and complete-cycle status.

- Heat-pump function: net work input provides the declared high-temperature heating duty.
- Thermal-interaction structure: isothermal under the framework-specific two-interaction classification.
- Heat-transfer reversibility: irreversible at both interfaces because the local differences are 10 K and 5 K.
- Profile-shape mismatch: absent because both members of each pair have $dT/dQ = 0$; finite-separation heat-transfer irreversibility remains.
- Complete-cycle reversibility: not established; the schematic does not model the internal components.

> **CONDUCTANCE NOTE**
> The constant approaches can be reduced under a comparison that allows the relevant operating temperatures to adjust. Chapter 6 distinguishes reductions attributable to increased conductance from Thermal-Matching interventions.

5.2.3 Fully Isothermal B/B Representation

The fully isothermal case is now shown in the B/B format used for the later canonical comparisons. Figure 8 makes the constant finite gaps visible while keeping both paired trajectories horizontal.

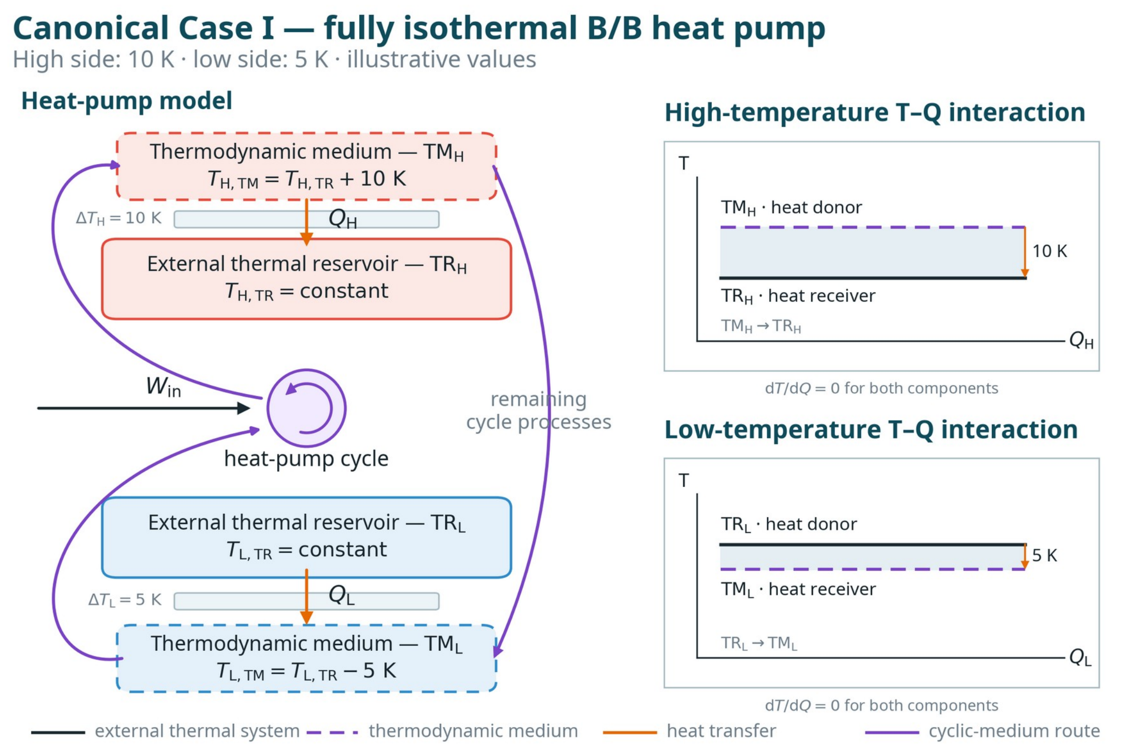


Figure 8. *Fully isothermal B/B paired-interface representation. Both paired trajectories are horizontal. Their finite 10 K and 5 K separations classify the two heat-transfer processes as irreversible while the interaction structure remains isothermal. Temperatures are illustrative.*

Source: author's own elaboration.

READING RULE
The absence of profile-slope mismatch does not mean the heat transfer is reversible. Reversibility requires removal of the constant vertical offset throughout each complete heat-transfer process.

5.3 Canonical Case II - External-Source Glide

Only the low-temperature external thermal source changes from isothermal to a linear glide.

5.3.1 Model definition

Canonical Case II introduces a low-temperature external-source glide while the thermodynamic medium remains isothermal.

$$T_{L,\mathrm{ETS}}(q) = 20\,°\mathrm{C} - 10\,\mathrm{K}\,q, \qquad T_{L,\mathrm{TM}} = 5\,°\mathrm{C} \tag{9}$$

$$\Delta T_L(q) = 15\,\mathrm{K} - 10\,\mathrm{K}\,q, \qquad \Delta T_{L,min} = 5\,\mathrm{K} \tag{10}$$

The external water-stream example enters the evaporator region at 20 °C and leaves at 10 °C, while the thermodynamic medium remains at 5 °C throughout the modelled low-temperature heat-receiving stage. The external source therefore reaches the same 10 °C terminal level used by the fully isothermal case, but its earlier heat increments are supplied at higher temperatures.

5.3.2 What remains unchanged

The thermodynamic-medium high- and low-stage temperatures are the same as in Canonical Case I. If the entire internal state path, heat quantities, component performance and all other model conditions are also held fixed, an internal-cycle COP may remain the same. The change in the external source trajectory alone does not prove equality of actual system COP, external pumping work, heat-exchanger performance or total plant performance.

5.3.3 Classification

The low-side interaction is therefore non-isothermal because the external source glides while the thermodynamic-medium trajectory remains isothermal.

- Continuously non-isothermal low-side interaction under the stated linear model.
- Finite and varying local temperature difference: the heat-transfer process is irreversible.
- Trajectory mismatch: external-source slope is nonzero while the thermodynamic-medium slope is zero.
- A 5 K terminal approach does not represent the larger differences present earlier in the interaction.

5.3.4 External-Source-Glide B/B Representation

Figure 9 applies the same B/B format after only the low-side external system is changed to a linear glide. This isolates a non-isothermal external-source trajectory against an isothermal thermodynamic-medium branch.

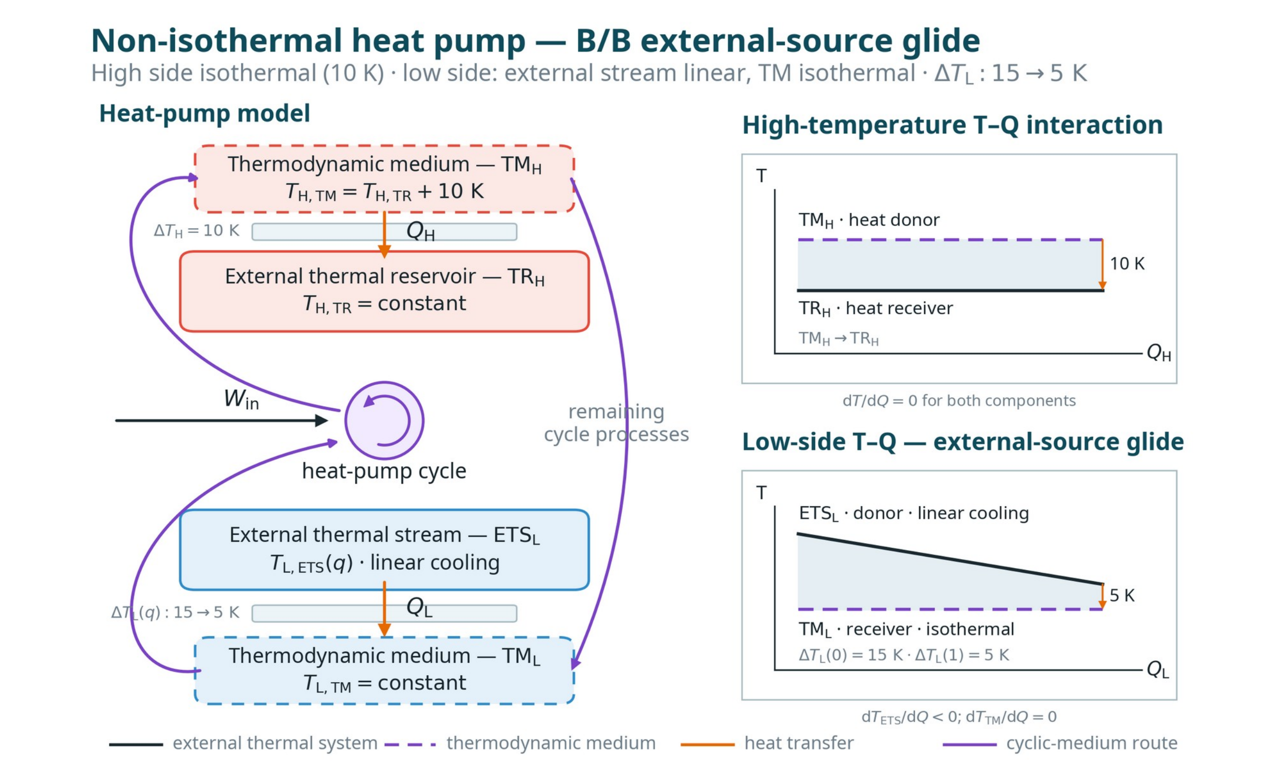


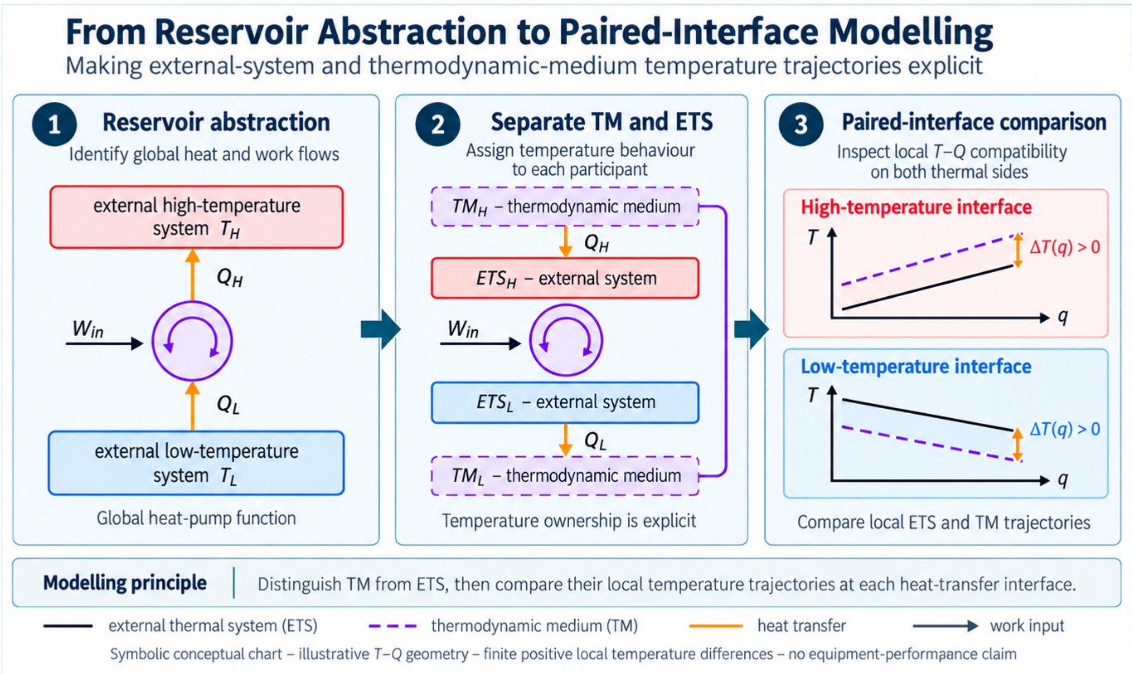
Figure 9. *B/B representation with a linearly cooling low-temperature external thermal source and an isothermal thermodynamic medium. The high side is unchanged. The low-side gap decreases from 15 K to 5 K.*
Source: author's own elaboration.

TERMINOLOGY NOTE
The gliding participant is represented as a finite-capacity external thermal stream rather than an ideal constant-temperature reservoir.

5.3.5 Interpreting a Linear T–Q Glide in Physical Space

A *T*–*Q* trajectory describes the thermodynamic temperature response of a participant to transferred heat: it shows how much the participant temperature changes when a given amount of heat is removed or supplied. For a finite sensible-heat mass, $Q = m\ c_p\ \Delta T$. For a steady stream such as water, the corresponding rate form is $\dot{Q} = \dot{m}\ c_p\ \Delta T$. When c_p and the relevant mass or mass-flow rate are approximately constant, the temperature varies linearly with accumulated heat transfer, so the straight line in the *T*–*Q* diagram represents a constant temperature change per unit of transferred heat.

This thermodynamic heat–temperature behaviour should not be confused with the spatial temperature profile along a physical heat exchanger. In a *T*–*x* representation, the local heat-transfer rate depends on both the local conductance and the local temperature difference. If the local conductance is approximately uniform, a larger local ΔT produces a larger local heat flux, so the sensible-heat participant changes temperature more rapidly over that spatial region. Near a smaller temperature difference or pinch, the spatial temperature change becomes more gradual. A trajectory that is linear in *T*–*Q* may therefore correspond to a curved trajectory in *T*–*x*.



Conceptual overview. The chart provides a reader-facing map from reservoir abstraction to explicit TM/ETS identification and local paired-interface comparison; Chapters 3 and 4 develop the architectures explicitly. The Formal Core separately provides the corresponding reversible-cycle construction and benchmark-relative lost/excess-work framework.

Source: author's own elaboration.

Figure 9a. *Same schematic heat-transfer interaction shown in T–Q and qualitative T–x coordinates. The left panel shows a linear sensible-heat glide opposite an isothermal participant, with a zero end pinch. The right panel illustrates how the same thermodynamic heat–temperature relation can correspond to a curved spatial temperature profile when the local heat-transfer rate varies with local temperature difference, assuming comparable local conductance. The figure is qualitative and is not a complete heat-exchanger design model, a finite-conductance optimum, or a literal reversible-limit construction.*

Source: author's own elaboration.

> **INTERPRETATION RULE**
> Linear in T–Q does not imply linear in T–x. The first describes temperature change per transferred heat; the second describes temperature distribution over physical distance.

5.4 Canonical Case III - Thermodynamic-Medium Glide

> *The low-temperature external source is isothermal; the thermodynamic medium heats linearly while receiving heat.*

5.4.1 Model definition

Canonical Case III moves the low-side glide from the external system to the thermodynamic medium.

$$T_{L,\mathrm{ETS}} = 10\,°\mathrm{C}, \qquad T_{L,\mathrm{TM}}(q) = -5\,°\mathrm{C} + 10\,\mathrm{K}\,q \qquad (11)$$

$$\Delta T_L(q) = 15\,\mathrm{K} - 10\,\mathrm{K}\,q, \qquad \Delta T_{L,min} = 5\,\mathrm{K} \tag{12}$$

The thermodynamic medium begins the modelled low-temperature heat-receiving stage at -5 °C, well below the 10 °C external source. It rises linearly to 5 °C. Only at the end does it reach the same 5 °C medium temperature used throughout the low-temperature stage of Canonical Cases I and II.

5.4.2 Consequence for the internal cycle

Unlike Canonical Case II, the thermodynamic-medium trajectory has changed. The internal heat-receiving path, state evolution and possibly the compression/expansion requirements therefore differ. COP must not be assumed unchanged merely because one terminal medium temperature, one minimum approach and the complete high side coincide with the earlier cases.

5.4.3 Classification

The low-side interaction is again non-isothermal, now because the thermodynamic-medium trajectory varies while the external source remains isothermal.

- Continuously non-isothermal low-side interaction under the stated linear medium model.
- Finite and varying local temperature difference: the heat-transfer process is irreversible.
- Trajectory mismatch: the external-source slope is zero while the medium slope is nonzero.
- The 5 K terminal approach is shared with the other cases, but the process-wide temperature history is not.

5.4.4 Thermodynamic-Medium-Glide B/B Representation

Figure 10 reverses the source of the glide relative to Case II: the external system remains isothermal while the thermodynamic-medium branch changes temperature. This makes the changed internal thermodynamic-medium path explicit.

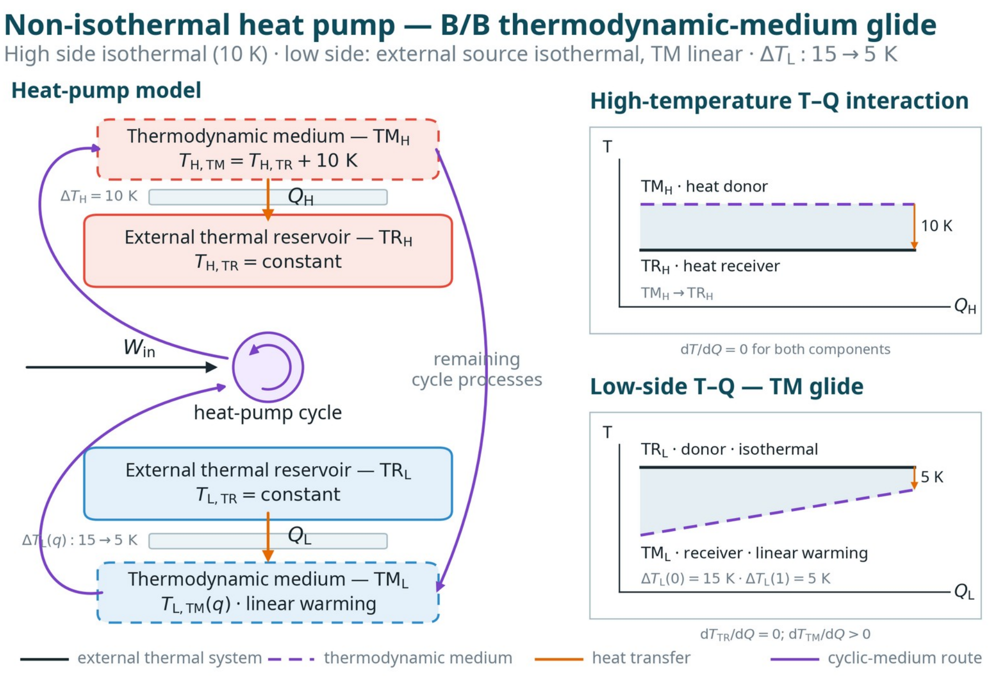


Figure 10. *B/B representation with an isothermal low-temperature external source and a linearly warming thermodynamic medium. The low-side gap decreases from 15 K to 5 K. The changed medium trajectory means COP is not assumed unchanged.*

Source: author’s own elaboration.

MODEL BOUNDARY

The linear TM trajectory is a schematic thermodynamic-medium boundary-temperature path. It is not identified with a specific pure or zeotropic refrigerant, pressure profile or phase-equilibrium calculation.

Next — cross-case comparison. The three canonical cases are now defined on a common basis. Section 5.5 compares them directly to show why the same terminal or minimum temperature approach can conceal fundamentally different process-wide temperature histories.

5.5 Cross-Case Comparison

> *The same terminal or minimum temperature approach can coexist with fundamentally different process-wide temperature histories. A single terminal ΔT therefore does not identify whether the external thermal system, the thermodynamic medium, or both are non-isothermal, nor does it establish equivalent cycle behaviour.*

Table 5. *Cross-Case Comparison*

Criterion	Fully isothermal	External-source glide	TM glide
High side	Both constant; 10 K gap	Same	Same
Low external system	10 °C constant	20 → 10 °C linear	10 °C constant
Low TM	5 °C constant	5 °C constant	-5 → 5 °C linear

Criterion	Fully isothermal	External-source glide	TM glide
Minimum low-side gap	5 K	5 K at end	5 K at end
Maximum low-side gap	5 K	15 K at start	15 K at start
Low-side profile-shape mismatch	No slope mismatch	Yes	Yes
Low-side heat transfer	Irreversible	Irreversible	Irreversible
Internal TM path vs Case I	Reference path	Unchanged by definition	Changed
COP equality vs Case I	Reference	Conditional only	Not assumed
Can UA alone remove retained profile-shape mismatch?	No retained profile-shape mismatch	No, under fixed one-flat/one-glide model	No, under fixed one-flat/one-glide model
Process-wide low-side temperature history	constant–constant	glide–constant	constant–glide
What the shared 5 K minimum/terminal gap does not show	5 K gap throughout	earlier gap up to 15 K; ETS path changes	earlier gap up to 15 K; TM path changes

COMPARISON PRINCIPLE

A comparison is valid only after the useful duty, system boundary, heat quantities, terminal conditions, area, flow conditions, component model and adjustable variables have been declared. Equal endpoints do not make two process trajectories thermodynamically equivalent. Equal terminal approaches do not imply equal local $\Delta T(Q)$ distributions, equal trajectory shapes, equal thermodynamic-medium state paths, or equal cycle-level work consequences.

Next — conductance and Thermal Matching. The cross-case differences show why local temperature-gap reduction, conductance enhancement and Thermal Matching must be distinguished. Chapter 6 develops that distinction and extends it to finite-conductance and dual-glide cases.

06 | Thermal Matching

> *This chapter distinguishes conductance effects from Thermal-Matching effects. It shows when a reduced temperature difference can be attributed to higher conductance alone and when it instead reflects a changed trajectory shape or temperature level. That distinction is essential if later improvement claims are to remain thermodynamically and engineering-wise well qualified.*

6.1 Overall Heat-Transfer Coefficient and Conductance

The overall heat-transfer coefficient U is an intensive coefficient. The product UA is the heat-transfer conductance and includes the heat-transfer area. For a declared steady two-stream approximation, the familiar relation $Q = UA\ \Delta T_{lm}$ may be useful, but it does not determine which temperatures, flows or duties change when U or A is changed.

$$\dot{Q} = U\,A\,\Delta T_{\mathrm{lm}} \tag{13}$$

A statement that higher U or UA reduces temperature differences is conditional: useful heat duty must be fixed while the comparison permits relevant operating variables to adjust. If heat duty and all inlet and outlet temperatures are fixed, increasing UA does not by itself move the plotted T–Q trajectories. Figures 11 and 12 isolate conductance effects; Figures 13–15 then change trajectory compatibility.

6.2 Conductance Enhancement of Profile-Matched Isothermal Trajectories

Figure 11 isolates conductance enhancement for two already profile-matched isothermal trajectories. At fixed heat duty, and with an operating temperature level permitted to adjust, increasing UA reduces the constant temperature separation required to transfer that duty. Both trajectories remain horizontal throughout: their shapes and slopes do not change.

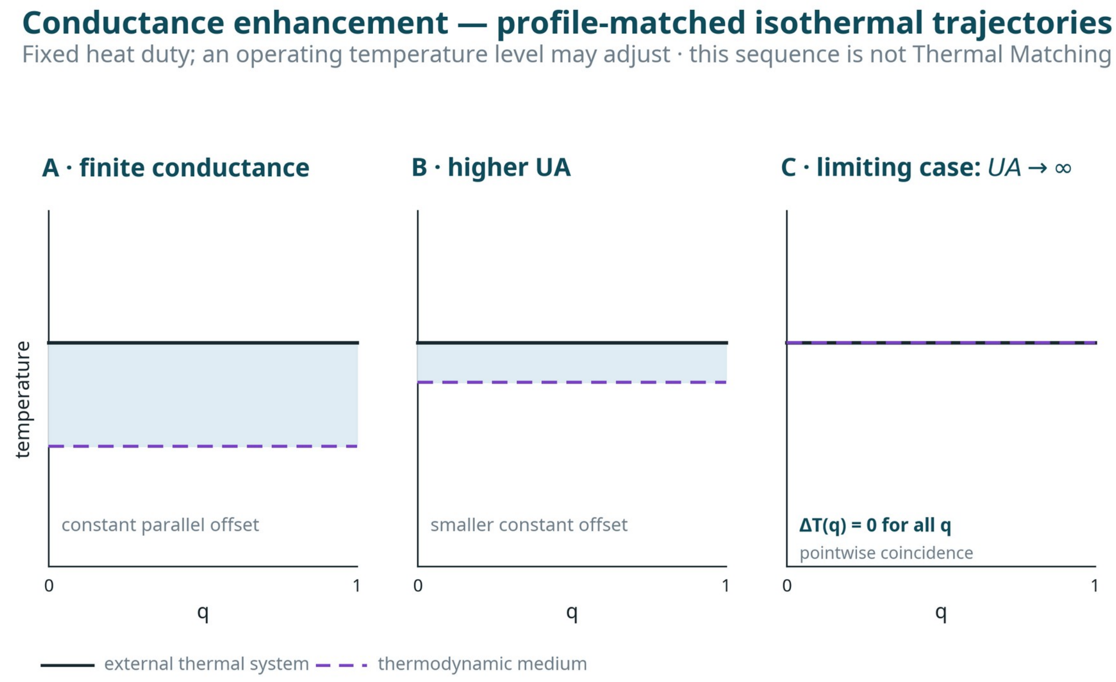


Figure 11. *Conductance enhancement of two already profile-matched isothermal trajectories. At finite conductance the profiles have a constant parallel separation; at higher UA the separation is smaller. In the theoretical limit UA tends to infinity, ΔT(q) tends to zero for every q and the profiles coincide pointwise. This sequence is conductance enhancement, not Thermal Matching, because the temperature-gap reduction is attributed solely to increasing UA; no independent trajectory-level, shape or heat-duty-distribution adaptation is imposed. The operating-temperature level is assumed able to adjust; if duty and all terminal temperatures are fixed, increasing UA does not by itself move the plotted profiles.*

Source: author's own elaboration.

CONDUCTANCE ENHANCEMENT — NOT THERMAL MATCHING

Here the paired isothermal trajectories are already shape-matched; their reduced parallel offset results solely from increased conductance. The UA→∞ endpoint is a theoretical reversible heat-transfer limit, not a realizable finite design value.

6.3 Thermal Matching: Direction-Neutral Definition

Thermal Matching is the proposed framework-specific engineering method for characterising and improving the local alignment of the thermodynamic-medium and external-system boundary-temperature trajectories at one or both heat-transfer branches. It may act on trajectory shape, temperature level, heat-duty distribution, or combinations of these, provided that the resulting configuration reduces the relevant local temperature differences under a stated comparison basis without relying solely on conductance enhancement. The method is therefore not restricted to non-isothermal interactions. Non-isothermal cases add trajectory-shape matching as an additional design degree of freedom. Candidate interventions include medium selection, mixture composition, pressure levels, mass flows, heat-capacity rates, phase-change behaviour, flow arrangement, staging and heat-duty distribution.

Operationally, Thermal Matching seeks to modify the paired T(Q) trajectories so that a greater fraction—or, where feasible, the complete heat-transfer duty—is transferred across smaller relevant local donor-to-receiver temperature differences $\Delta T(q)$, without introducing an inadmissible temperature crossover. Under a thermodynamically qualified like-for-like comparison, reducing these local temperature differences moves the affected heat-transfer process toward its reversible limit and reduces the thermodynamic irreversibility attributable to that heat-transfer process.

SCOPE OF THERMAL MATCHING
Thermal Matching concerns compatibility of the paired thermodynamic-medium and external-system boundary-temperature trajectories through changes in temperature level, trajectory shape, heat-duty distribution, or combinations of these. It is distinct from conductance enhancement and does not quantify or remove internal-cycle irreversibilities such as compressor or expander inefficiency, throttling, pressure loss, friction, mixing, leakage or electrical losses.

6.4 Operational Assessment of Thermal Matching

Thermal Matching is assessed against a declared comparison basis. The complete paired temperature trajectories matter: one terminal approach, one pinch point or a slope comparison alone is insufficient when the local gap varies through the transferred-heat coordinate.

A compact assessment should therefore state: (1) what function, duty, boundary and terminal conditions are held fixed; (2) which physical variable or trajectory is changed; (3) the before-and-after paired T–Q trajectories; (4) the resulting local, minimum, maximum and terminal temperature differences; and (5) any COP or work-input consequence only when the cycle model is sufficiently specified.

Within the stated comparison basis, a Thermal-Matching change is favourable when improved alignment of trajectory shape, temperature level, heat-duty distribution, or combinations thereof reduces the relevant local temperature differences without introducing an inadmissible temperature crossover, and when the improvement is not attributable solely to conductance enhancement. At finite conductance, equal dT/dQ slopes are neither necessary nor sufficient for a thermodynamic optimum; Figures 13 and 14 therefore illustrate only one possible slope-adjustment route and do not prescribe which trajectory must change or what the finite-conductance optimum must look like.

THERMODYNAMIC CONSEQUENCE — The principal thermodynamic objective is not geometric alignment for its own sake. Smaller relevant local $\Delta T(q)$ values mean less departure of the affected heat-transfer process from its reversible limit. Under an isolating comparison in which the other irreversible mechanisms are held fixed, this means less thermodynamic irreversibility and therefore a smaller cycle-level work penalty: less benchmark-relative lost work for a heat engine, or less excess work input for a heat pump or refrigerator. This is a genuine thermodynamic performance gain. It does not, however, require a proportional change in the conventional ratio metrics Wout/Qin, QH/Win or QL/Win; those ratios depend on how their numerator and denominator change within the declared comparison class.

RELATION TO THE FORMAL CORE
The paired T–Q geometry diagnoses thermal mismatch at the interfaces; it does not by itself give a cycle-level work penalty. Chapter 7 states the two quantification levels. The Formal Core supplies the corresponding-reversible-cycle construction and benchmark-relative lost/excess-work method.

6.5 Conductance Enhancement with One-Sided Glide

Figure 12 isolates conductance enhancement while the one-sided glide shape is held fixed. It therefore shows what higher conductance can reduce and what it cannot remove when a trajectory mismatch remains.

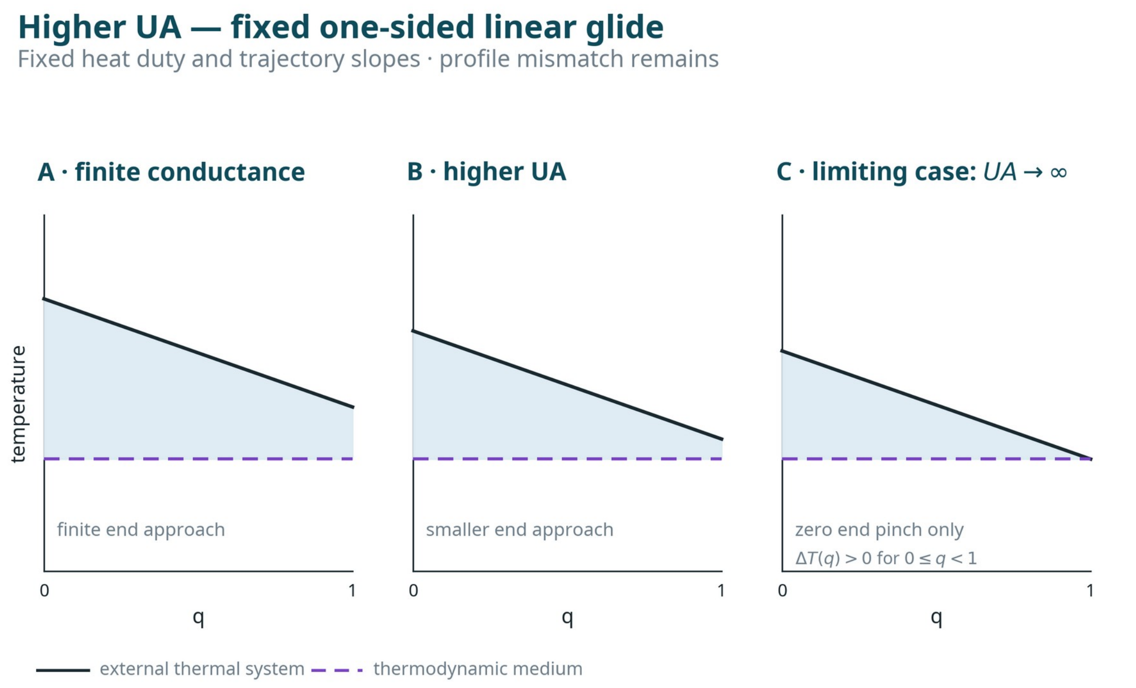


Figure 12. *Conductance enhancement applied to a fixed one-sided linear temperature glide. At finite and higher UA, the terminal approach decreases while the slope mismatch is unchanged. In the limiting case UA tends to infinity, a zero end pinch occurs at $q = 1$, but a finite process-wide gap remains for $0 \leq q < 1$. Increasing conductance alone therefore cannot remove the glide mismatch.*
Source: author's own elaboration.

CONDUCTANCE LIMIT
When unequal slopes are retained, even the ideal UA-to-infinity limit can create only a terminal zero pinch. It cannot make the two temperature trajectories coincide over the complete heat-transfer duty.

6.6 Progressive Thermal Matching - Vanishing-Separation Limit

Figure 13 then changes the trajectories themselves rather than conductance alone. The sequence is a constructed path toward pointwise coincidence and is used only to illustrate one possible Thermal-Matching route.

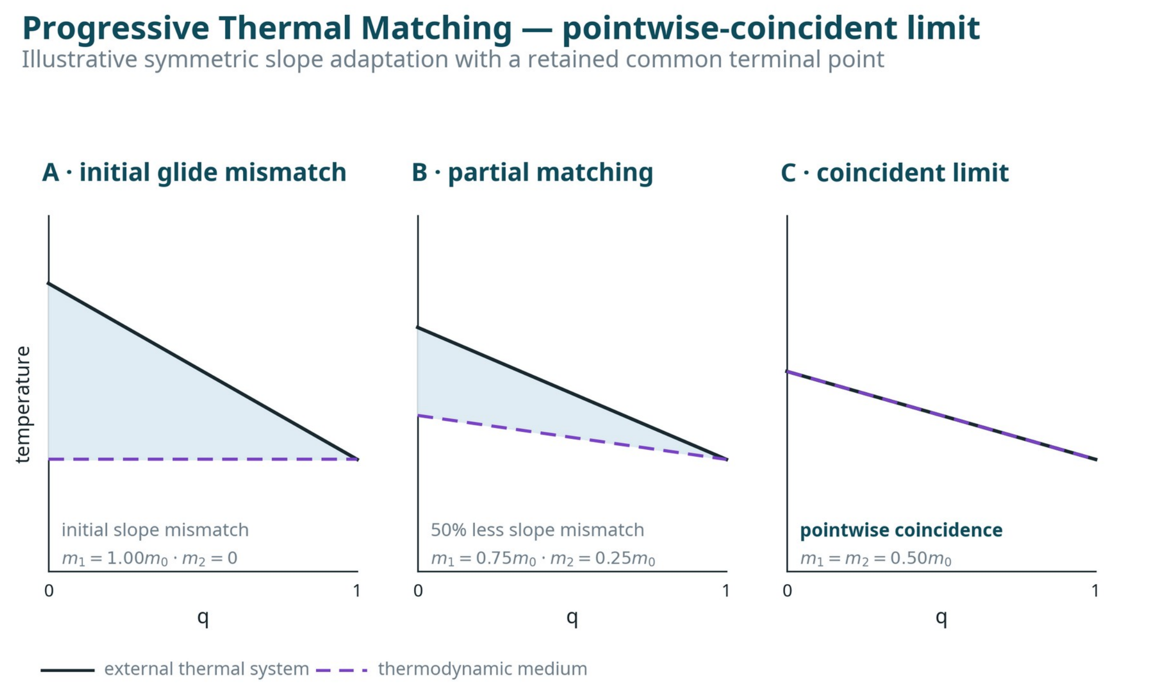


Figure 13. *One illustrative route for progressive Thermal Matching with a retained common terminal point. For visual symmetry, both trajectories are modified: in the middle panel, each slope is shifted by 25% of the initial slope difference, reducing the slope mismatch by 50%. In the final panel both slopes equal 0.50 times the initial glide and the profiles coincide pointwise. This is the vanishing-local-separation heat-transfer limit; the symmetric adjustment is not a definitional requirement.*

Source: author's own elaboration.

PROFILE MATCHING AND REVERSIBLE LIMIT

In this constructed sequence, slope mismatch and remaining offset vanish together because the profiles share a common endpoint. Thermal Matching does not require symmetric movement or prescribe which participant is the fixed reference.

6.7 Progressive Thermal Matching at Finite UA

Figure 14 keeps finite conductance while the displayed slope mismatch is progressively reduced. It separates improved profile compatibility from the still-required positive local driving temperature difference.

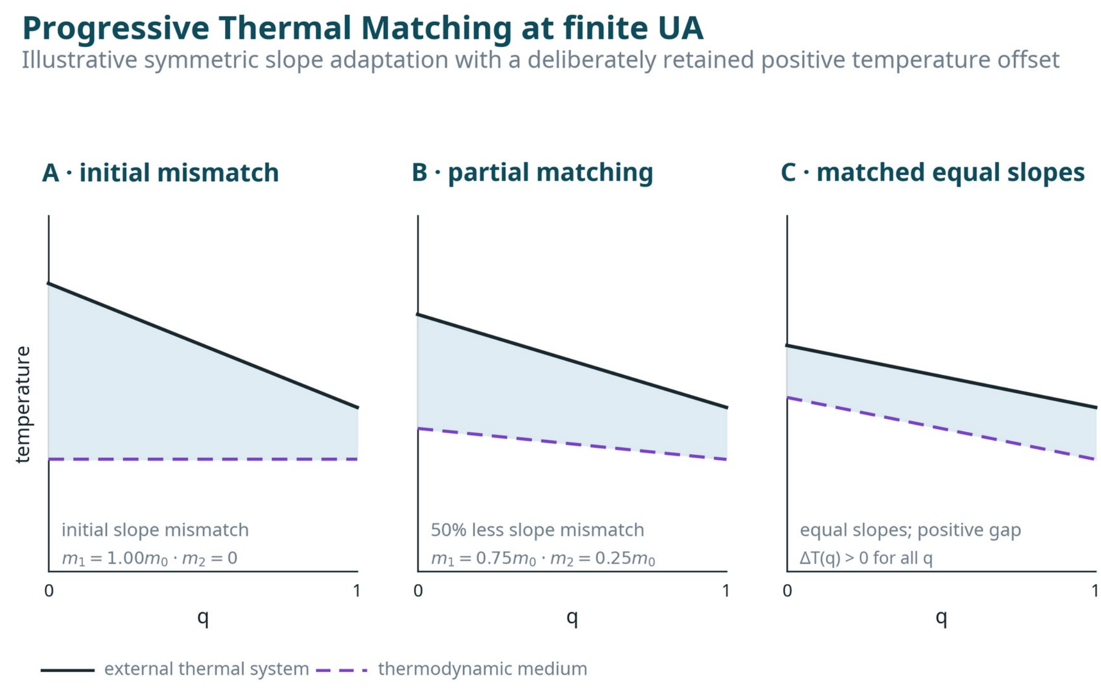


Figure 14. *The same illustrative two-trajectory slope-adaptation sequence at finite UA. The upper trajectory remains visibly displaced above the lower trajectory. In the final panel the slopes are equal and the profile-shape mismatch is removed, but a finite parallel temperature gap remains. Heat transfer therefore remains thermodynamically irreversible. The retained offset and equal-slope endpoint are conditions of this example, not a general finite-conductance optimum criterion.*

Source: author's own elaboration.

At finite conductance, equal slopes remove the illustrated shape mismatch but do not define a general thermodynamic optimum. Depending on the declared constraints and adjustable variables, a preferred configuration may retain non-uniform local temperature differences and unequal slopes.

THERMAL MATCHING BOUNDARY

The retained parallel offset is a condition of this example, not a restriction on Thermal Matching. Other interventions may change temperature levels, trajectory shapes or heat-duty distribution; none addresses irreversibilities elsewhere in the cycle.

6.8 Engineering Synthesis — Dual-Glide Thermal Matching at Finite Conductance

The preceding cases isolate individual thermal-interaction features. This synthesis combines them in one heat-pump representation in which both the thermodynamic medium and the external thermal system follow linear non-isothermal trajectories at both the high- and low-temperature interfaces. The comparison is intentionally qualitative and contains no equipment-performance or optimum calculation.

Configuration A represents the initial dual-glide mismatch. Configuration B retains both external-system trajectories while adapting the thermodynamic-medium trajectories so that the absolute TM–ETS slope mismatch is reduced by 50% at both interfaces. The relevant local temperature differences are also reduced while remaining positive. Configuration B therefore represents improved Thermal Matching at finite conductance; it is not identified as the thermodynamic optimum or as the reversible limit.

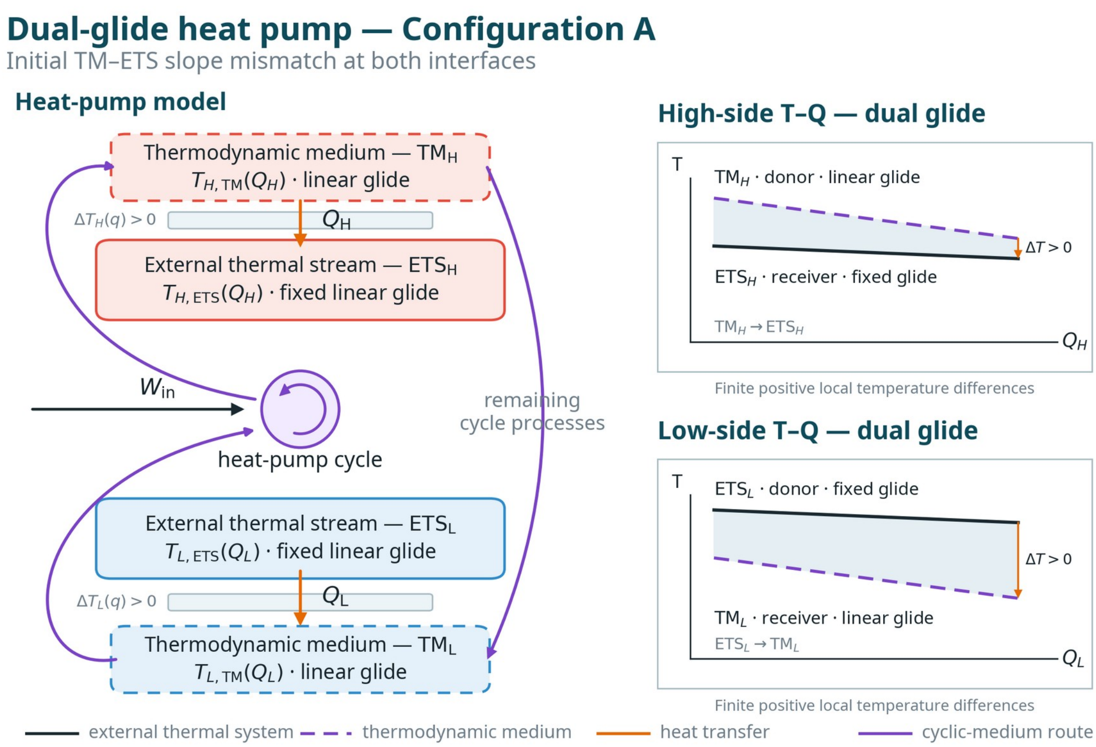


Figure 15a. *Engineering synthesis — Configuration A. Both participants follow linear non-isothermal T–Q trajectories at the high- and low-temperature interfaces. The displayed TM–ETS slope mismatch and finite positive local temperature differences define the initial dual-glide condition. Geometry is illustrative; no equipment-performance or optimum claim is made.*

Source: author's own elaboration.

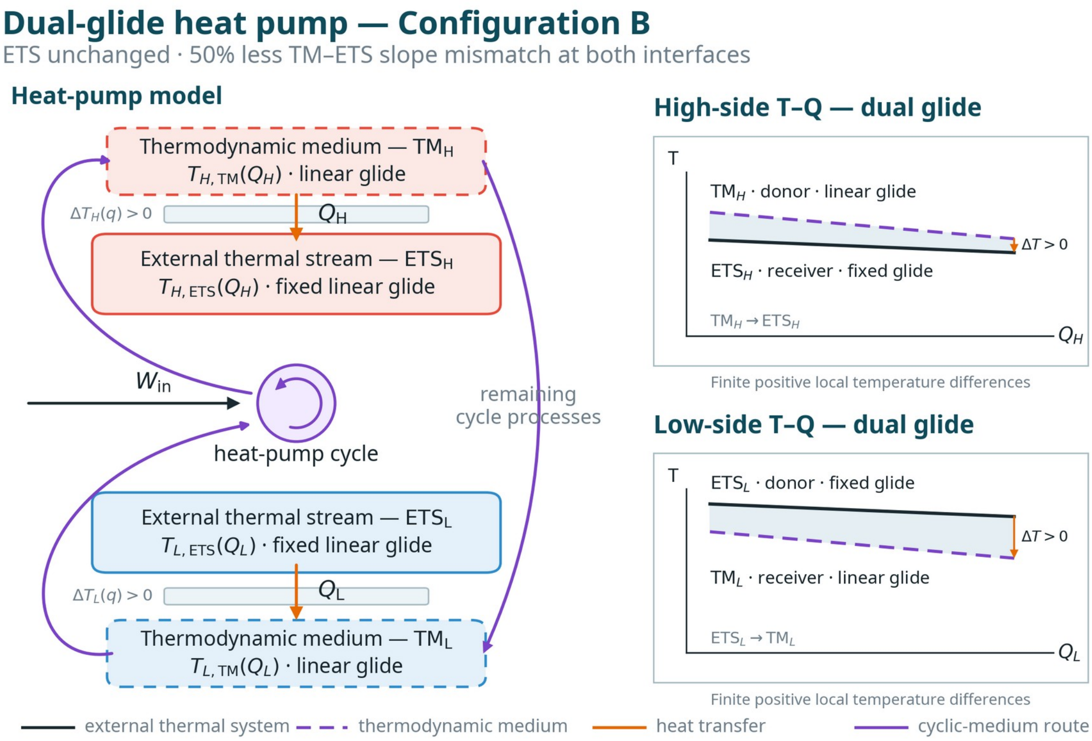


Figure 15b. *Engineering synthesis — Configuration B. The external-system trajectories are retained while the thermodynamic-medium trajectories are adapted so that the absolute TM–ETS slope mismatch is reduced by 50% at both interfaces. The local temperature differences are reduced but remain positive. The figure illustrates improved Thermal Matching at finite conductance, not a thermodynamic optimum or the reversible limit.*

Source: author's own elaboration.

ENGINEERING INTERPRETATION

Thermal Matching can act simultaneously at both external heat-transfer interfaces. At finite conductance, improved alignment may coexist with unequal slopes, non-uniform local temperature differences and irreversible heat transfer. The preferred configuration depends on the declared cycle function, useful duty, boundary conditions, conductance and other engineering constraints.

Next — quantification levels. Chapter 6 has established how Thermal Matching changes the paired interfaces. Chapter 7 separates this interface-level diagnosis from the complete-cycle quantification of the associated thermodynamic work consequence.

07 | Quantification Levels, COP and Work-Input Implications

This chapter explains what can be quantified directly from a paired-interface diagram and what requires a fuller cycle model. It separates interface-level diagnosis from complete-cycle thermodynamic performance assessment, including COP and work-input implications. By keeping those quantification levels distinct, the report avoids reading more into a schematic T–Q representation than the declared data actually justify.

$$\mathrm{COP}_H = \frac{Q_H}{W_{\mathrm{in}}}, \qquad \mathrm{COP}_L = \frac{Q_L}{W_{\mathrm{in}}} \tag{14}$$

A B/B figure supplies essential heat-temperature information but not a complete cycle model. A numerical COP additionally requires heat quantities, internal state relations, component behaviour, pressure levels, mass flows, closure and the selected useful-duty definition.

7.1 Two Levels of Quantification

This companion distinguishes two complementary levels of quantification. The first characterises Thermal Matching or Thermal Mismatching directly at the paired heat-transfer interfaces. The second quantifies the thermodynamic work consequence at complete-cycle level. The two levels are physically related but are not interchangeable.

Table 6. *Two Levels of Quantification*

Level	What is quantified	Primary representation / output
Interface level	The distribution and change of Thermal Matching or Thermal Mismatching at an individual high- or low-temperature heat-transfer interface.	Paired T–Q trajectories; local $\Delta T(Q)$; minimum, maximum and terminal approaches; trajectory shape and slope differences; crossover check. These are interface diagnostics, not a scalar thermodynamic magnitude of irreversibility.
Cycle level	The complete-cycle work consequence of irreversibility relative to the corresponding reversible cycle under the declared function, boundary, useful duty and comparison conditions.	Benchmark-relative lost work for a heat engine or excess work input for a heat pump or refrigerator within the Formal Core method. This aggregate quantity does not by itself allocate the penalty to one heat-transfer interface or other local mechanism.

At interface level, the paired T–Q trajectories, local $\Delta T(Q)$, terminal approaches and trajectory-shape or slope differences describe where thermal mismatch occurs, how it is distributed over the transferred-heat coordinate, and how it changes between configurations. They do not constitute a standalone scalar quantification of the thermodynamic magnitude of irreversibility produced by that individual heat-transfer interface. At cycle level, the complete irreversible cycle is compared with its corresponding reversible cycle. That comparison yields the aggregate benchmark-relative work consequence for the declared comparison class.

Benchmark-relative thermodynamic improvement and conventional ratio improvement are therefore related but not identical. Within a fixed and valid comparison class, a reduction in lost work or excess work input is an unambiguous reduction of the thermodynamic work penalty, even when the conventional heat-engine efficiency Wout/Qin or heat-pump/refrigerator COP ratio does not increase. The detailed reasons and governing equations are retained in the Formal Core rather than repeated in this companion.

QUANTIFICATION BOUNDARY
Finite boundary-temperature differences at one or both external heat-transfer interfaces establish heat-transfer irreversibility and are retained as local diagnostic information. In this companion, their thermodynamic magnitude is not quantified by assigning a separate lost-work value to an individual interface. The work consequence is quantified from the complete-cycle comparison between the irreversible cycle and its corresponding reversible cycle. Attribution of a change in lost or excess work to one or both finite-temperature-difference heat-transfer processes is valid only when the comparison class holds all other irreversible mechanisms fixed or excludes them. If other mechanisms change simultaneously, the resulting lost or excess work is their combined cycle-level consequence.

7.2 Conditional Comparison Statements

The three canonical cases isolate different changes in the paired heat-transfer trajectories, but the T–Q diagrams alone do not determine cycle-level COP or work input. The following statements are therefore conditional: they apply only when the relevant cycle model, heat quantities and comparison conditions are specified consistently.

- Fully isothermal case: reducing both external approach temperatures can reduce required temperature lift and work input when the cycle and control model permit the medium levels to move accordingly.
- External-source-glide case: the internal thermodynamic-medium trajectory may remain identical to the fully isothermal case, but equality of system COP is permitted only when Q_H, Q_L, W_{in} and all other relevant internal and auxiliary conditions are also identical.
- Thermodynamic-medium-glide case: the internal heat-receiving path changes; COP is therefore recalculated rather than inherited from either previous case.
- For every case, improving only the two modelled thermal interfaces does not remove compressor, expansion, frictional, pressure-drop or auxiliary-work penalties.

ILLUSTRATIVE HEAT-ENGINE READING — A Thermal-Matching gain need not appear as fewer joules of supplied thermal energy. Under an appropriately defined comparison, the same heat quantity and useful-work target may instead be associated with a less demanding high-temperature heat-supply trajectory. The thermodynamic improvement is then expressed in the Q–T specification of the supplied heat and in a reduced benchmark-relative work penalty, while Wout/Qin may remain unchanged. This example is interpretive; validity requires a consistent comparison class, and the detailed derivation belongs to the Formal Core.

7.3 Relation to the Corresponding Reversible Cycle

This is the second quantification level. Within the broader Formal Core method, the complete irreversible cycle is compared with its corresponding reversible cycle under the same declared function, system boundary, useful duty and comparison conditions. The associated cycle-level work consequence is expressed as benchmark-relative lost work for a heat engine or excess work input for a heat pump or refrigerator. This quantity is aggregate at the selected cycle boundary: it does not automatically allocate the total to a particular heat-transfer interface, compressor, pressure loss or other irreversible mechanism. To attribute a change specifically to one or both finite-temperature-difference heat-transfer processes, the comparison class must hold the remaining irreversible mechanisms fixed or exclude them. This technical companion does not reproduce the formal cycle construction or calculation.

Next — generalized assessment protocol. With interface-level diagnostics and cycle-level quantification separated, Section 7.4 consolidates the method into a reusable Thermal-Matching assessment protocol.

7.4 Generalized Thermal-Matching Assessment Protocol

This section consolidates the preceding analysis into a reusable assessment protocol for future cases. It lists the declarations and checks needed to describe a paired-interface Thermal-Matching case in a disciplined and comparable way. The protocol is intentionally compact, but it preserves the distinctions on boundary conditions, entities, trajectories and admissible interpretation developed earlier in the report.

The protocol below converts the preceding physical and thermodynamic distinctions into a repeatable assessment and reporting sequence. Its purpose is not to prescribe a unique cycle architecture or optimisation route, but to ensure that future cases declare the same minimum information before Thermal Matching, COP or work consequences are interpreted. The steps should therefore be read as a consistency and disclosure sequence: first establish cycle function, boundary, participants and directions; then define and plot the paired temperature trajectories and classify the local heat-transfer behaviour; finally state the comparison basis, closure conditions and epistemic status.

1. Declare the cycle function: heat pump for useful high-temperature duty or refrigerator for useful low-temperature duty.
2. Define the cyclic-system boundary and identify the single thermodynamic medium and all external thermal systems.
3. Classify the high and low interactions independently as isothermal, piecewise-isothermal, continuously non-isothermal or mixed.
4. Assign donor and receiver roles and define Q_H, Q_L, W_{in} and the sign convention.
5. Select a common local Q or normalized q coordinate for each paired interface.
6. Specify both boundary-temperature trajectories and state whether each is measured, calculated, assumed or illustrative.
7. Plot both trajectories and report local, minimum, maximum and terminal temperature differences where relevant.
8. Classify heat-transfer irreversibility separately from complete-cycle reversibility.

9. Distinguish conductance enhancement from Thermal Matching; within Thermal Matching distinguish temperature-level, trajectory-shape and heat-duty-distribution changes where relevant.
10. State the comparison class before drawing COP, work-input or optimisation conclusions. If a change in cycle-level lost or excess work is attributed to one or both external heat-transfer processes, state explicitly which other irreversible mechanisms are held fixed or excluded.
11. Verify First-Law closure, constitutive sufficiency, process/cycle closure, admissibility and architecture claims.
12. Report epistemic status, source basis, limitations and validation status.

Minimum future figure set. Every new canonical case should include one B/B architecture diagram, one high-side T–Q panel, one low-side T–Q panel, one compact assumption block and one status sentence covering interaction structure, finite-temperature-difference irreversibility, the relevant Thermal-Matching degree of freedom, and COP qualification.

08 | Cross-Application Example — Brayton-Type Gas-Turbine Heat Engine

> *This chapter shows that the paired-interface method is not limited to the heat-pump examples used as the main worked line of argument. Using a simplified Brayton-type heat-engine recognition case, it demonstrates how the same temperature-ownership and trajectory logic carries over to another thermodynamic cycle family. For a heat-pump specialist, the gas-turbine example is a directly related Q–T engineering analogue: the simplified Brayton branches and water–water heat-pump interfaces both contain isothermal–linear paired-interface geometries, although the gliding participant and heat/work directions differ. The chapter then qualifies how that interface logic transfers—and where it does not transfer—to direct combustion, open exhaust flow, recuperation and downstream heat use.*

Reader orientation. This chapter can be read as a bounded stand-alone cross-application example. The underlying paired-interface logic is developed earlier in the report: Chapter 3 introduces the heat-engine counterpart and paired-interface principle; Chapter 4 transfers that architecture to heat pumps; Chapter 5 separates cycle function, thermal-interaction structure and reversibility; and Chapter 6 defines Thermal Matching in terms of deliberate improvement of paired T(Q) trajectories and the relevant local donor-to-receiver temperature differences ΔT(q) under a stated comparison basis. Cycle-level work consequences and benchmark-relative lost or excess work are treated separately in Chapter 7. Readers already familiar with these principles may proceed directly with the Brayton-type application below.

8.1 Purpose and modelling boundary

In the illustrative air-standard representation used here, compressed thermodynamic medium enters the high-temperature heat-addition branch at a relatively low temperature and warms through an approximately linear T–Q glide as heat is supplied. After expansion through the turbine/expander, the hot thermodynamic medium enters the low-temperature branch and cools through an approximately linear T–Q glide while rejecting heat. The high-temperature source and low-temperature sink are shown as isothermal only to provide a controlled first-order paired-interface reference.

For a gas-turbine reader, the high-temperature external reservoir in this abstraction represents externalized heat addition associated with combustion, while the low-temperature reservoir is an idealized environmental sink used to close the air-standard cycle. Neither object should be read literally as the corresponding real-engine process. In a directly fired gas turbine, fuel enters as a reacting mass flow, chemical composition changes through combustion, and the exhaust of an open engine leaves as a mass flow rather than through a required low-temperature heat exchanger. Sections 8.3–8.5 therefore distinguish the air-standard recognition model from the more qualified interface interpretations needed for real gas-turbine applications.

8.2 Paired-interface Brayton-type representation

Figure 16 makes the two idealized thermodynamic-medium heat-transfer branches explicit. At the high-temperature side, the externalized source is isothermal while the thermodynamic medium warms non-isothermally; at the low-temperature side, the thermodynamic medium cools non-isothermally while the idealized external sink remains isothermal. Finite positive local temperature differences are retained at both branches, so the displayed heat-transfer processes are irreversible even though the external source and sink are isothermal. The figure is a recognition model for paired-interface geometry, not a literal representation of a directly fired combustor or bare open exhaust.

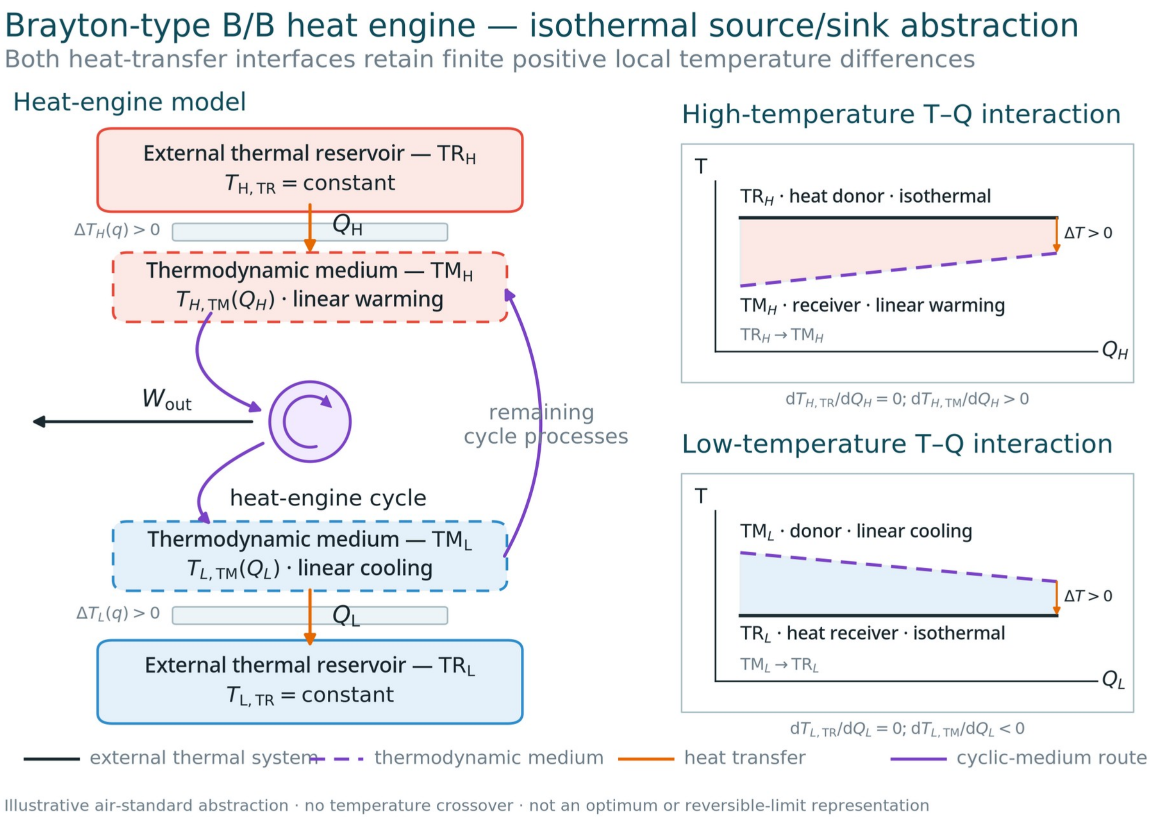


Figure 16. *Brayton-type B/B heat-engine recognition case. The high-temperature heat-addition branch is represented by an isothermal external source paired with a linearly warming thermodynamic medium; the low-temperature heat-rejection branch pairs a linearly cooling thermodynamic medium with an isothermal external sink. Both branches retain finite positive local temperature differences. The figure is an illustrative air-standard abstraction: it is not a complete combustor or open-cycle gas-turbine model, not a thermodynamic optimum, and not a reversible-limit construction.*

Source: author’s own elaboration.

CYCLE-FUNCTION EQUIVALENCE — At the paired-interface Q–T level, the thermodynamic classification is direction-neutral. Isothermal versus non-isothermal and reversible versus irreversible remain independent classifications for heat engines and heat pumps; cycle function changes the heat/work directions and performance metric, not the interface-analysis grammar. Section 3.3 shows why finite-ΔT irreversibility nevertheless prevents exact simultaneous preservation of both medium and external temperature trajectories under cycle reversal.

CROSS-DOMAIN Q–T RELEVANCE — The Brayton-type interfaces represented here and the water–water heat-pump interfaces discussed earlier both contain the same descriptive isothermal–linear pairing: one temperature trajectory is approximately isothermal while the other follows an approximately linear glide. Which participant owns the glide can differ, but the same local Q–T engineering questions—temperature ownership, ΔT(q), profile compatibility, Thermal Matching and approach to the reversible local limit—apply. This correspondence concerns the heat-transfer branches and does not imply equivalence of combustion, compressor/turbine, refrigerant, pressure, control or equipment-design problems.

8.3 High-temperature side — combustion-side qualification and Thermal-Matching degrees of freedom

In a directly fired gas turbine, fuel enters the combustor as a reacting mass flow rather than as an external thermal reservoir. The air-standard replacement of combustion by external heat addition is therefore a useful cycle abstraction, but it does not preserve the mass transfer, chemical reaction, mixing, changing composition and property evaluation required for a real combustor. A combustion-side energy-release trajectory is consequently not automatically identical to a boundary heat-transfer trajectory, and there is no unique combustion-source temperature trajectory unless a physical or mathematical temperature attribution has first been defined.

Within suitably defined local control volumes, chemical-energy conversion, local energy-release rates and the associated thermodynamic-medium temperature development can be represented at progressively finer spatial or temporal resolution. Such a representation does not turn combustion into conventional heat transfer between two physically separated bodies. Any proposed paired comparison must state the control-volume boundary, the reacting mixture represented, the energy-release quantity being accumulated along Q or q, and the rule by which a temperature is assigned to the combustion side. The more general term thermal-availability trajectory should be used only where that quantity is separately defined and cannot be confused with established exergy or availability terminology.

For such a diagnostic, the horizontal Q or q coordinate must represent an explicitly accumulated process quantity—for example, a declared portion of chemical-energy release or thermodynamic-medium heating—not a thermodynamic state property. The two plotted quantities need not be physically identical objects merely because they share a common heat coordinate. Their comparison is meaningful only when the mapping between local energy release, local medium heating and the assigned temperature information is stated well enough that each point on one trajectory can be paired with the corresponding increment on the other. This is precisely where the direct-combustion case differs from a conventional heat exchanger: the pairing itself is part of the proposed model and must be operationalized before Thermal Matching can be evaluated.

Subject to those qualifications, a temperature-resolved comparison between a constructed combustion-side energy-release trajectory and the thermodynamic-medium heating trajectory may be useful as an interface-oriented Thermal-Matching diagnostic. In this report that construction is a proposed methodological interpretation and research direction, not an established combustor model and not an independently validated route to higher gas-turbine efficiency or lower fuel use.

Potential trajectory degrees of freedom for a future detailed combustor model include staged or sequential fuel injection, distribution of fuel between combustion zones, local fuel–air ratio, fuel evaporation and reaction characteristics, mixing and dilution, residence time, cooling-air addition, pressure evolution, and the spatial and temporal distribution of energy release. These variables may reshape the relation between energy release and thermodynamic-medium heating. They also alter other parts of the combustor and cycle, so any apparent improvement in trajectory matching must be assessed together with pressure loss, reaction and mixing irreversibility, cooling demand, emissions and stability constraints, and compressor/turbine performance. Staged combustion or energy-release shaping is therefore treated here as a candidate design and research direction, not as a demonstrated net-performance improvement.

8.4 Low-temperature side — exhaust boundary and configuration

The low-temperature trajectory is boundary-dependent. Direct exhaust discharge, atmospheric cooling, recuperation, a heat-recovery steam generator, a bottoming cycle, process-heat delivery and district-heating service are thermodynamically different configurations and should not be collapsed into one generic environmental-sink representation. The idealized sink in Figure 16 belongs to the air-standard closure of the cycle; it is not a claim that every open gas turbine contains a physical low-temperature heat-transfer interface.

For a bare open gas-turbine engine without recuperation, bottoming-cycle recovery or another exhaust heat exchanger, energy and entropy leave the selected engine control volume primarily with the outgoing exhaust mass flow. A conventional paired heat-transfer-interface representation is obtained only after the system boundary is extended to include an explicit downstream thermal interaction, such as exhaust cooling against another stream or useful heat delivery. Each such case must therefore identify whether the governing boundary is the engine, the complete plant, or an extended downstream heat-recovery system.

This distinction is also important when the exhaust ultimately cools to ambient conditions. The fact that an exhaust stream will eventually exchange energy with the environment does not by itself make that downstream cooling process part of the engine cycle boundary. If atmospheric cooling, a stack model or another finite downstream cooling path is introduced as part of the thermodynamic problem, its temperature trajectory and boundary definition must be stated explicitly. Until then, the appropriate low-side description for the bare engine is an outgoing mass-flow state, not an assumed paired reservoir-to-medium heat-transfer branch.

When the external environment is effectively fixed, its temperature trajectory normally offers little direct design freedom. The gas-side exhaust trajectory can nevertheless change through pressure ratio, expansion and exhaust state, turbine and nozzle work distribution, cooling and mixing flows, and the addition or removal of recuperation or downstream heat recovery. These changes affect the complete cycle and cannot be interpreted as isolated low-temperature-interface adjustments. Their interface-level Q–T consequences can be displayed locally, while their net work, efficiency and fuel consequences require the complete engineering comparison.

8.5 Recuperation and downstream heat-use configurations

Recuperation provides the most direct gas-turbine example of the paired-interface method because the hot turbine-exhaust stream and the cooler compressed-air stream remain physically distinct across a heat exchanger. In conventional air-standard gas-turbine treatments this component is often called a regenerator; in recuperator form the streams exchange heat through a separating wall. Their complete T(Q) trajectories can therefore be compared directly in terms of local temperature difference, pinch location, heat-capacity-rate mismatch and departures from linear behaviour caused by changing gas properties.

This physical separation makes recuperation methodologically more direct than the constructed combustion-side comparison. The donor and receiver are identifiable streams, transferred heat passes through a defined heat-exchanger boundary, and the local donor-to-receiver temperature difference has the conventional heat-transfer meaning used elsewhere in this report. In an idealized regenerative Brayton treatment, exhaust heat preheats the compressor discharge and can reduce the external high-temperature heat addition required for the specified cycle states. In a real recuperator, pressure losses, finite effectiveness, material temperature limits and off-design behaviour can partly offset that benefit, so the paired T(Q) diagnosis remains one layer of the complete design assessment rather than a stand-alone performance predictor.

For Thermal Matching, the relevant engineering question is how the two stream trajectories can be made more compatible over the transferred-heat coordinate while maintaining an admissible positive local driving temperature difference at finite conductance. Heat-exchanger effectiveness, conductance, pressure loss, size, mass, cost and operating-range constraints remain complementary design variables rather than components of the trajectory-matching definition itself. Established gas-turbine analysis already recognizes regeneration as a means of using hot turbine exhaust to preheat compressor discharge; the paired T(Q) view adds an explicit interface-level description of where thermal mismatch occurs along that exchange.

The same trajectory analysis can be applied, with configuration-specific boundaries, to heat-recovery steam generators, combined- or bottoming-cycle heat recovery, process-heat service, cogeneration, district-heating interfaces and other forms of exhaust-heat utilization. These cases are noted here only as distinct paired-interface configurations. Their component models, plant integration and optimization are outside the present report.

8.6 Established gas-turbine engineering, Brayton relations and methodological contribution

Gas-turbine R&D and industry already contain extensive empirical, experimental and operational knowledge for adapting real engines and plants. Established practice includes fuel-flexible and multi-fuel/dual-fuel combustion, combustor tuning and control, adaptation to changing fuel properties, operating-profile and load adjustment, component and gas-path upgrades, regeneration/recuperation, heat recovery, and broader cycle or plant optimisation. None of these established engineering practices is claimed as new here.

Established Brayton relations describe idealized cycle performance under their stated assumptions and remain essential for gas-turbine analysis. The paired T(Q) treatment used in this report addresses a different but complementary question: it makes the heat–temperature trajectories and local thermal mismatch at explicitly modelled interfaces visible rather than replacing aggregate Brayton performance relations. A broader cycle-level solution requires First-Law closure together with applicable medium properties, pressure and process constraints, compression and expansion relations, state definitions, architecture choices and admissibility conditions. Alternative well-posed Brayton solution forms are reserved for a future cycle-level treatment; they are not derived here.

The same separation applies to optimization. A cycle-level Brayton optimization may vary pressure ratio, turbine inlet state, component efficiencies, recuperation and other variables simultaneously, whereas an interface-level Thermal-Matching study asks a narrower question about the paired heat–

temperature trajectories produced by a declared configuration. The two analyses can inform one another, but they are not interchangeable. A trajectory change that reduces local thermal mismatch may alter pressure levels, state points, mass flows or heat duties elsewhere in the cycle; conversely, a cycle optimization can change the paired T(Q) trajectories even when Thermal Matching was not its explicit objective.

The methodological contribution assessed in this companion is therefore bounded: a common paired-interface Q–T representation is applied across heat-pump and gas-turbine cases, while the combustion-side trajectory is retained as a qualified proposed research direction. The method can organise interface-level temperature ownership, local ΔT(q), trajectory compatibility and Thermal-Matching questions, but any claim of improved net engine performance, fuel consumption or emissions requires a complete gas-turbine model and separate validation.

8.7 Interface-level versus cycle-level scope and future gas-turbine treatment

The paired-interface analysis developed here diagnoses where and how thermal mismatch occurs at explicitly modelled heat-transfer interfaces. For a real open gas turbine, that diagnostic may apply directly to a recuperator or downstream heat-recovery exchanger, and only conditionally to a constructed combustion-side energy-release trajectory whose temperature attribution and control volume have been defined. A bare exhaust mass flow is not converted into a heat-transfer interface merely by drawing an environmental sink.

Construction of a corresponding reversible cycle and quantification of the complete-cycle work consequence require energy closure and the applicable reversible-cycle, process, state and constitutive relations. Local T(Q) trajectories and ΔT(q) diagnose interface-level Thermal Matching; they do not independently assign lost work to one interface. Any cycle-level attribution must use a qualified comparison class that holds other irreversible mechanisms fixed or explicitly includes their combined effect.

Accordingly, the present chapter stops at the level needed to transfer the paired-interface methodology responsibly to recognizable gas-turbine configurations. It does not provide a full reacting-flow combustor model, chemical-exergy treatment, detailed open-cycle benchmark construction, or alternative Brayton derivations.

FUTURE GAS-TURBINE TREATMENT

A separate gas-turbine volume can examine real non-isothermal combustion and reacting-flow energy release, explicit definitions for any combustion-side source or availability trajectory, open-cycle boundaries, fuel and gas-property effects, cooling and mixing flows, emissions and stability constraints, detailed recuperator and heat-recovery integration, and the relationship between established Brayton-cycle relations and the programme's proposed operationalized Clausius benchmark framework. Full reversible-cycle construction, alternative cycle-level solution forms and quantitative performance validation remain outside this Thermal Matching companion.

09 | What Thermal Matching Adds

This chapter synthesizes the main added value of the paired-interface method. It states what becomes newly visible when thermodynamic-medium and external-system trajectories are represented explicitly, what kind of engineering decisions Thermal Matching can inform, and what limits remain. In that way, the chapter gathers the document's practical message without overstating what the present report alone establishes.

9.1 Core Contributions of Thermal Matching

Paired-interface visibility. Conventional reservoir-only representations leave the thermodynamic-medium side of each heat-transfer interface implicit. Making both TM and ETS trajectories explicit reveals temperature ownership, local driving differences and process-wide trajectory behaviour.

Thermal Matching as an engineering degree of freedom. Thermal Matching improves local thermal alignment through deliberate changes in temperature level, trajectory shape, heat-duty distribution or combinations of these. Improvement attributable solely to increased heat-transfer conductance remains conductance enhancement. Non-isothermal interactions add trajectory-shape matching as a further design degree of freedom.

Finite-conductance and reversible limits. A better finite-conductance configuration need not have equal slopes, uniform local temperature differences or pointwise coincidence. The reversible local heat-transfer limit is the stronger condition in which the paired boundary-temperature trajectories coincide pointwise over the complete interaction.

Two-level assessment. Paired T–Q trajectories and local $\Delta T(Q)$ diagnose Thermal Matching or Thermal Mismatching at interface level. The thermodynamic work consequence is quantified at complete-cycle level by comparing the irreversible cycle with its corresponding reversible cycle; formal lost-work or excess-work-input quantification remains in the Formal Core.

Proposed benchmark hierarchy. In classical thermodynamics and gas-turbine engineering, the Carnot benchmark is commonly treated as the generic reversible-cycle reference. The Formal Core presents a derivation that appears to falsify this conventional hierarchy at the level of benchmark generality. Under the proposed operational framework, the Clausius reversible-cycle equality provides the generic parent relation, while the Carnot reversible-cycle equality and the familiar Carnot efficiency and COP relations are recovered exactly as its two-isothermal specialization. The Clausius reversible-cycle equality—not the familiar Carnot formulas—therefore forms the proposed parent framework for generic reversible-cycle benchmarking. This does not invalidate the Carnot relations within their proper domain; it reclassifies their place within the proposed hierarchy. The conclusion remains subject to independent scientific review and validation. The associated research communication and planned independent-validation pathway are summarized in the closing Scientific Source Basis, Status and Rights Notice.

This companion has demonstrated the method primarily with heat pumps and added a Brayton-type gas-turbine heat-engine cross-application example. The same paired-interface logic is applicable to heat engines, refrigeration cycles and other thermodynamic cycles with externally coupled heat-transfer processes. Broader application opportunities are addressed in the Impact, Applications &

Research Opportunities Report; the formal corresponding-reversible-cycle construction and cycle-level lost/excess-work quantification are developed in the Formal Core.

9.2 Educational and Textbook Context — Representative Moran Assessment

Temperature-profile analysis, pinch analysis and temperature-glide matching are established in specialist heat-transfer, heat-pump and refrigeration literature. In the bounded textbook corpus examined for the CarnotX publication programme, no complete treatment was identified that combines explicit thermodynamic-medium/external-system trajectories, the conductance-versus-mismatch distinction, process-specific reversible benchmarking and cycle-level work consequences.

The observation is cross-domain: the examined heat-pump/refrigeration and gas/power-cycle treatments contain established component and cycle relations, but did not consolidate them into the same cycle-level Thermal-Matching workflow used in this report.

Formal Core Appendix D therefore retains a representative assessment of Moran et al., Moran's Principles of Engineering Thermodynamics, 9th edition, SI/Global Edition (2018). It identifies no fundamental correction to the established Carnot or Clausius relations; the issue assessed is the operational and educational architecture.

MORAN ASSESSMENT — BOUNDED RESULT
Conditional on scientific acceptance of the proposed operationalization, the bounded assessment identifies 18 distinct modernization intervention types: 10 necessary and 8 desirable, with no established formula correction or erratum identified. The detailed intervention map remains in Formal Core Appendix D.

The resulting modernization spans the Second-Law/Carnot–Clausius foundation, power and gas cycles, refrigeration and heat pumps, heat exchangers and refrigerant selection. For this companion, the relevant implication is the same explicit paired-interface T–Q treatment across both heat-engine and heat-pump domains.

EVIDENCE BOUNDARY
This representative case does not establish prevalence across all thermodynamics textbooks, editions or curricula. It supports only the bounded conclusion that the integrated cycle-level treatment used here was not identified as standard architecture in the examined corpus.

Appendix A - Canonical Equations and Illustrative Data

> *The table below consolidates the illustrative boundary-temperature data used in the three canonical cases. It is provided as a compact reference for the figures and direct diagnostics, not as an additional equipment calculation.*

Table 7. *Canonical Equations and Illustrative Data*

Item	Fully isothermal	External-source glide	TM glide
High donor TM	60 °C constant	60 °C constant	60 °C constant
High receiver ETS	50 °C constant	50 °C constant	50 °C constant
High ΔT	10 K constant	10 K constant	10 K constant
Low donor ETS	10 °C constant	20 - 10q °C	10 °C constant
Low receiver TM	5 °C constant	5 °C constant	-5 + 10q °C
Low ΔT	5 K constant	15 - 10q K	15 - 10q K
Low dTETS/dq	0	-10 K	0
Low dTTM/dq	0	0	+10 K

A.1 Governing direct diagnostics

The expressions below collect the direct interface diagnostics used throughout the report without introducing a separate scalar irreversibility index.

$$\Delta T_j(q) = T_{\text{donor},j}(q) - T_{\text{receiver},j}(q)$$

$$\text{reversible thermal interface} \Leftrightarrow \Delta T_j(q) = 0 \text{ for every } q$$

$$\text{linear profile slope: } \frac{dT}{dQ} = \frac{T_{\text{out}} - T_{\text{in}}}{Q_{\text{tot}}}$$

All temperatures used in thermodynamic ratios must be absolute temperatures in kelvin. Degrees Celsius are used only for the illustrative diagram labels because temperature differences in kelvin and degrees Celsius have equal numerical increments. No Carnot or Clausius ratio is evaluated from the Celsius values on this page.

NO UNIVERSAL SCALAR INDEX

The local gap and its heat-coordinate distribution are retained as problem-specific interface information. This report introduces no universal scalar Thermal Matching or thermodynamic-irreversibility index and does not assign a standalone lost-work magnitude to an individual finite-temperature-difference interface. The thermodynamic work consequence is treated at complete-cycle level through the corresponding reversible-cycle comparison in the Formal Core; local attribution requires an isolating comparison class.

Appendix B - Diagram Production Specification

> *This appendix records the minimum visual rules required for future paired-interface figures. It supports consistency of ownership, direction, colour, captioning and acceptance checks across the report and related companion outputs. Its purpose is production discipline rather than new thermodynamic argument.*

B.1 Mandatory visual checks

The checks below define the minimum visual acceptance criteria for future paired-interface figures so that physical ownership, heat-flow direction and trajectory meaning remain unambiguous.

Table 8. *Diagram Production Acceptance Checks*

Check	Acceptance requirement
Entity separation	TM and ETS are separate labelled objects at both interfaces.
One medium	The high and low TM stages are visibly connected by one cycle route.
Heat direction	Q_L: $ETS_L \rightarrow TM_L$; Q_H: $TM_H \rightarrow ETS_H$.
Work direction	W_{in} points into the cyclic system.
Temperature ownership	Every T label identifies the physical side to which it belongs.
Common coordinate	The two trajectories of one interface use the same Q or q axis.
Line semantics	External thermal system solid; thermodynamic medium dashed; isothermal horizontal; glide sloped.
Gap annotation	State whether ΔT is constant, varying or a minimum/maximum/terminal value.
Qualification	Caption distinguishes illustrative geometry from calculated or validated performance.
Accessibility	Meaning is available from labels and line styles, not colour alone.

B.2 Preferred figure caption pattern

A figure caption should identify the physical configuration, state what the paired trajectories show, and qualify what the figure does and does not establish.

> **CAPTION TEMPLATE**
> Figure X. [Cycle function and interaction structure]. [Donor and receiver trajectories]. [Direction and local temperature-difference interpretation]. [What is omitted]. [Whether values are illustrative, calculated or validated]. Source: [declared source].

B.3 File-production recommendation

Maintain a vector master for each diagram and export a high-resolution publication asset for Word. Store the numerical trajectory data and equation definition alongside the figure so that a static publication graphic remains reproducible even when it is not edited directly as Word drawing shapes.

Scientific Source Basis, Status and Rights Notice

Scientific source basis

Governing programme sources. The governing scientific source is Formal Core R185-L18, Operationalizing the Carnot and Clausius Reversible-Cycle Equalities for Process-Specific Thermodynamic Benchmarks. The governing terminology and positioning source is Scientific Positioning Standard v4.2/R181. The visual and programme-positioning reference is Impact, Applications & Research Opportunities Report v1.8.

Established prior art and priority boundary. The Impact, Applications & Research Opportunities Report distinguishes established temperature-glide matching and T–Q profile analysis from the programme's proposed cycle-level integration. Contextual prior art includes Zühlsdorf et al. (2018), “Analysis of temperature glide matching of heat pumps with zeotropic working fluid mixtures for different temperature glides”, Energy 153, 650–660, doi:10.1016/j.energy.2018.04.048. The present report therefore does not claim priority for temperature-profile matching itself.

Proposed programme contribution. The proposed contribution is a physics-explicit paired-interface visual and operational framing within the CarnotX programme, connected to the Formal Core's proposed complete operationalization of the Clausius reversible-cycle equality for constructing corresponding reversible cycles and quantifying benchmark-relative lost or excess work. The three canonical model cases and their illustrative temperatures consolidate the author-directed working decisions developed for the present report.

Gas-turbine and heat-recovery sources. Established background for Chapter 8 is checked against the project-held Moran et al., Moran's Principles of Engineering Thermodynamics, 9th edition, SI/Global Edition (2018), including its air-standard Brayton-cycle treatment, regenerative gas-turbine heat-exchanger model, reacting-system control-volume treatment and heat-recovery steam-generator examples. In that source, the air-standard combustion temperature rise is represented by external heat transfer, the idealized Brayton closure uses heat rejection to restore the working fluid, a regenerator transfers heat from turbine exhaust to compressor discharge, and waste-heat-recovery examples explicitly distinguish stream-to-stream temperature-difference irreversibility. Representative industrial context includes GE Vernova documentation on power-plant flexibility, fuel-flexible and multi-fuel gas-turbine operation, combustion fuel flexibility and adaptive fuel-property control, together with Siemens Energy documentation on fuel-flexible and hydrogen-capable gas turbines, accessed September 2026.

Educational evidence boundary. Chapter 9.2 summarizes the bounded Moran textbook assessment retained in Formal Core Appendix D. The representative modernization assessment concerns Moran et al., Moran's Principles of Engineering Thermodynamics, 9th edition, SI/Global Edition (2018); related direct inspection in the source lineage includes Moran and Shapiro, Fundamentals of Engineering Thermodynamics, 5th edition, SI version (2006). The assessment is a representative case, not a systematic prevalence study, and edition-specific chapter, equation and wording references require direct verification against the licensed edition before external publication.

Scope retained outside this report. The complete formal cycle construction, work-penalty calculation and expanded treatment of established Brayton-cycle relations within the programme's proposed operationalized Clausius benchmark framework remain outside this technical companion. The gas-turbine treatment is reserved for the planned gas-turbine volume.

Recent Research and Validation Programme

Recent research programme. The Formal Core, Operationalizing the Carnot and Clausius Reversible-Cycle Equalities for Process-Specific Thermodynamic Benchmarks, documents the current derivations, thermodynamic qualification, worked calculation examples and application cases. Its central proposal is not a new thermodynamic law and does not replace the Clausius equality. Rather, it proposes a complete mathematical and thermodynamic operationalization of that established equality, combined with First-Law closure and explicit boundary, reversibility, admissibility, selection and reporting conditions. The proposed hierarchy treats the Carnot relations as the exact two-isothermal specialization within a broader Clausius-based construction framework. This remains a recent research conclusion requiring independent scientific review and validation.

Research communication. The public communication and learning channel for this programme is CarnotX Academy. It provides public orientation material, access to research documents, visual explanations and context for developing CarnotX software demonstrations. Publication status, software implementation, verification and independent validation remain separately identified.

Proposed independent validation. A multidisciplinary validation programme is being developed as a proposed Lorentz Center workshop in Leiden, with participation from technical-thermodynamics specialists and relevant researchers from Leiden University, particularly in physics, chemistry and astronomy, supported where useful by mathematics and computational science. The programme would reconstruct the derivations, expose assumptions and sign conventions, reproduce classical limiting cases, test broader reversible-cycle cases, and actively search for counterexamples or qualification limits. Until the workshop is accepted and the programme is completed, it is a planned validation pathway rather than external confirmation of the Formal Core conclusion.

Status

The table below summarises the status of the principal report elements and keeps proposed methods, illustrative examples and validation claims explicitly separated.

Table 9. *Scientific Status of Principal Report Elements*

Item	Classification
A/A and B/B architecture codes	Proposed report-specific visual method; the preceding reservoir-only abstraction is conventional and is used as the comparison baseline.
Thermodynamic distinctions	Established science and Formal Core interpretation as identified in the text
Three numerical examples	Illustrative schematic cases; not equipment simulations
Thermal Matching integration	Proposed paired-interface cycle-level framing aligned with the Formal Core Thermal Matching definition. Temperature-glide/profile matching itself is established in specialist engineering literature and is not claimed as new here. The connection to corresponding reversible-cycle construction and benchmark-relative lost/excess-work quantification belongs to the broader Formal Core method. "Isothermal–linear paired-interface class" is report-specific descriptive terminology, not established nomenclature.

Item	Classification
Brayton/gas-turbine cross-application	Illustrative air-standard recognition case plus bounded gas-turbine extensions. Established open-flow, reacting-system, regenerator/recuperator and heat-recovery concepts are separated from the proposed paired-interface interpretation. A constructed combustion-side energy-release trajectory remains a proposed research direction requiring explicit temperature attribution and control-volume definition. No net efficiency, fuel-saving or combustor-performance improvement is independently validated here.
Validation	Internal consistency and document QA only; no independent scientific or experimental validation claimed.

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STATUS BOUNDARY
Public availability and document production do not constitute independent thermodynamic review, equipment validation, publication acceptance or proof of design performance.